退職教員特別寄稿
【報告等】

京都工芸繊維大学学術報告書　第 4 巻（平成 22 年 11 月）
BULLETIN OF KYOTO INSTITUTE OF TECHNOLOGY 4（Nov 2010）

On a Synthesis of Multivariable PID Controllers for a Class of Nonlinear Systems

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（2010 年 3 月 30 日原稿受理、2010 年 7 月 22 日採用決定）

Abstract

This paper is concerned with a synthesis of multivariable proportional-integral-derivative (PID) controllers for a class of nonlinear systems. First, mathematical model of plant with sector-bounded nonlinearity and multivariable PID controller is given. Secondly, the closed-loop system is described by a descriptor form. A sufficient condition for checking the L_2-performance of descriptor systems with sector-bounded nonlinearity is introduced. Based on the condition, the problem of synthesizing multivariable PID controllers is formulated as a bilinear matrix inequality one. The problem is solved by an iterative linear matrix inequality algorithm to synthesize a PID controller so as to expand the region of sector and minimize L_2-gain. Finally, the synthesis method is applied to the synthesis of a PID controller for a ball-on-wheel system to verify the effectiveness.

Key Words: *multivariable PID controller; sector-bounded nonlinearity; descriptor system; matrix inequality; L_2-performacne.*

1. Introduction

It goes without saying that most of dynamical behavior of industrial apparatus is described by nonlinear mathematical models. Over the past few decades, the concept of sector-bounded nonlinearity has been used in order to analize and synthesize a class of nonlinear systems[1). In recent years, methods of analyzing and synthesizing the systems with sector-bounded nonlinearity via linear matrix inequality (LMI) have been vigorously studied, and applied to various nonlinear systems such as systems with input saturation[2). The convex optimization problem subject to LMIs can be solved efficiently by the interior-point method[3).

The proportional-integral-derivative (PID) control scheme has been widely used in various industrial control systems, and most of conventional PID controller synthesis methods are based on the relatively simple representation of plant dynamics such as the characterization by a single-input/single-output model. These are, however, not necessarily sufficient for plants with multi-input/multi-output.

Recently, multivariable PID controller design method in time domain[4),5),6),7)] and a pre-assigned structure-constrained PID controller design method[8)] in frequency domain have been proposed. However, these methods have dealt with only design for linear plant.

In this paper, a method of synthesizing multivariable PID controllers for systems with sector-bounded nonlinearity so as to minimize the L_2-gain and expand the region of sector is presented. Then, this paper extends the analysis condition proposed by F. Wu and B. Lu[2)].

The paper is organized as follows. In Section 2, mathematical model of nonlinear plant with multivariable PID controller is given. The analysis condition is obtained in order to synthesize multivariable PID controllers for systems with sector-bounded nonlinearity in Section 3. Section 4 is devoted to the formulation of our problem as an optimization one, which is subject to a bilinear matrix inequality (BMI). In Section 5, multivariable PID controller is first obtained for a linearized plant, and then, the controller for nonlinear system is determined so as to expand the region of the sector and minimize the L_2-gain by an iterative LMI algorithm. In Section 6, the synthesis method is utilized for synthesizing a PID controller for a ball-on-wheel system to illustrate the effectiveness.

The principal symbols used in this paper are listed below:

$\mathbf{R}^n$	:	set of all real n-vectors
$\mathbf{R}^{m\times n}$	:	set of all real $m \times n$-matrices
I_n	:	$n \times n$-identity matrix
A^T	:	transpose of a vector or matrix A
$A > (\geq) 0$	:	A is positive (semi-)definite
$A > (\geq) B$	:	$A - B$ is positive (semi-)definite
$\mathrm{diag}\,(a_1, \ldots, a_p)$	:	diagonal matrix whose elements are $a_1, \ldots, a_p$
$\mathrm{tr}(A)$	:	trace of matrix A

$\|A\|_\infty$: H_∞ norm of matrix A defined by $\|A\|_\infty := \sup_{\omega\in\mathbf{R}} \sigma_{\max}(A(j\omega))$

$\|z\|_{L_2}$: L_2 norm of vector $z(t)$ defined by $\left\{\int_0^\infty z(t)^T z(t)\, dt\right\}^{1/2}$.

2. Problem Statement

Consider a time-invariant nonlinear plant described by

$$\begin{cases} \dot{x}_p(t) = A_p x_p(t) + B_{pu}u(t) + B_{pv}v(t) + B_{pw}w(t) \\ \tilde{v}(t) = C_{vp}x_p(t) \\ y(t) = C_y x_p(t) \\ z(t) = C_{zp}x_p(t) + D_{zu}u(t) + D_{zw}w(t), \end{cases} \tag{1}$$

and

$$v(t) = \psi(\tilde{v}(t)), \tag{2}$$

where $x_p(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^\ell$, $y(t) \in \mathbf{R}^r$ and $z(t) \in \mathbf{R}^m$ $(\ell, r \leq n)$ are the plant state, control input, measured output and controlled output, respectively. $w(t) \in \mathbf{R}^s$ is the exogenous disturbance such that $\|w\|_{L_2} < \infty$, and A_p, B_{pu}, B_{pv}, B_{pw}, C_{vp}, C_y, C_{zp}, D_{zu} and D_{zw} are constant matrices of appropriate dimensions. The matrices B_{pu} and C_y are assumed to be of full column rank and full row rank, respectively, i.e., $\mathrm{rank}\, B_{pu} = \ell$ and $\mathrm{rank}\, C_y = r$.

The vector-valued function $\psi : \mathbf{R}^p \to \mathbf{R}^p$ is a nonlinear function which is assumed to belong to

$$\Delta = \{\psi : v = \psi(\tilde{v}),\ (v - H_1\tilde{v})^T W (v - H_2\tilde{v}) \le 0\}, \tag{3}$$

where $\tilde{v} \in \mathbf{R}^p$, $H_1 = \mathrm{diag}\,(h_{11}, h_{12}, \ldots, h_{1p})$, $H_2 = \mathrm{diag}\,(h_{21}, h_{22}, \ldots, h_{2p})$, and $W = \mathrm{diag}\,(W_1, W_2, \ldots, W_p) > 0$.

The relation between the nonlinear function ψ and the sector $\{(\tilde{v}, v) : (v - H_1\tilde{v})^T W (v - H_2\tilde{v}) \le 0\}$ for given H_1, H_2 and W is shown in Fig. 1. Throughout this paper, the slope matrix H_1 is fixed for simplicity.

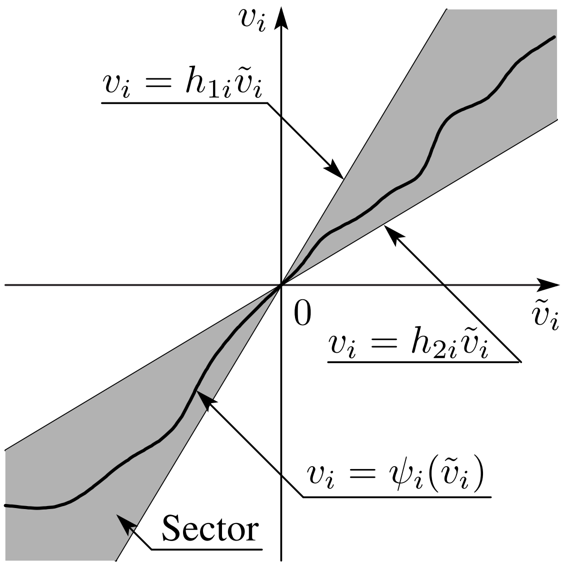

Fig. 1. Relation between nonlinear function and sector.

The control input $u(t)$ is generated by the following multivariable PID controller :

$$u(t) = K_P y(t) + K_I \int_0^t y(\tau) d\tau + K_D \dot{y}(t), \tag{4}$$

where K_P, K_I and K_D are the PID controller gain matrices of appropriate dimensions.

In this paper, we develop a method of synthesizing PID controller (4) so as to minimize γ and $\mathrm{tr}\,(H_2)$, where γ is an index in the performance condition $\|z\|_{L_2} \leq \gamma\|w\|_{L_2}$, and the minimization of $\mathrm{tr}\,(H_2)$ stands for the enlargement of the sector.

3. Analysis of a Class of Descriptor Systems with Sector-bounded Nonlinearities

In order to synthesize PID controller (4) for systems with sector-bounded nonlinearity in Δ, we introduce the following proposition obtained by extending the result by F. Wu and B. Lu[2)].

Proposition: Assume that a scalar $\gamma > 0$, diagonal matrices H_1 and H_2 and coefficient matrices E, A, B_v, B_w, C_v, C_z and D_w in the system

$$\begin{cases} E\dot{x}(t) = Ax(t) + B_v v(t) + B_w w(t) \\ \tilde{v}(t) = C_v x(t) \\ z(t) = C_z x(t) + D_w w(t), \end{cases} \tag{5}$$

are given. If there exist a nonsingular matrix $X_{c\ell}$ and positive-definite diagonal matrix $W > 0$ such that

$$E^T X_{c\ell} = X_{c\ell}^T E \geq 0 \tag{6}$$

$$\begin{bmatrix} A^T X_{c\ell} + X_{c\ell}^T A + H(W; H_2) & X_{c\ell}^T B_v + C_v^T (H_1 + H_2) W & X_{c\ell}^T B_w & C_z^T \\ B_v^T X_{c\ell} + W(H_1 + H_2) C_v & -2W & 0 & 0 \\ B_w^T X_{c\ell} & 0 & -\gamma^2 I_s & D_w^T \\ C_z & 0 & D_w & -I_m \end{bmatrix} < 0, \tag{7}$$

where $H(W; H_2) = -C_v^T H_1 W H_2 C_v - C_v^T H_2 W H_1 C_v$, then, for all $\psi \in \Delta$, the system (5) with $v = \psi(\tilde{v})$ satisfies $\|z\|_{L_2} \leq \gamma\|w\|_{L_2}$ provided that $Ex(0) = 0$.

Proof: By the Schur complement formula[9], we see that (7) is equivalent to

$$\begin{bmatrix} A^T X_{c\ell} + X_{c\ell}^T A + H(W; H_2) & X_{c\ell}^T B_v + C_v^T (H_1 + H_2) W & X_{c\ell}^T B_w \\ B_v^T X_{c\ell} + W(H_1 + H_2) C_v & -2W & 0 \\ B_w^T X_{c\ell} & 0 & -\gamma^2 I_s \end{bmatrix} + \begin{bmatrix} C_z^T \\ 0 \\ D_w^T \end{bmatrix} \begin{bmatrix} C_z & 0 & D_w \end{bmatrix} < 0. \tag{8}$$

Multiplying (8) by $[x^T \ v^T \ w^T]$ on the left and by its transpose on the right and using (6), we have

$$\frac{d}{dt}(x^T E^T X_{c\ell} x) - 2(v - H_1 \tilde{v})^T W (v - H_2 \tilde{v}) - \ \gamma^2 w^T w + z^T z \leq 0. \tag{9}$$

For all $\psi \in \Delta$, (9) implies

$$\frac{d}{dt}(x^T E^T X_{c\ell} x) - \gamma^2 w^T w + z^T z \leq 0. \tag{10}$$

Integrating both sides of (10) from $t = 0$ to $t = t$, we have

$$\begin{aligned} &\int_0^t z^T(\tau) z(\tau) d\tau - \gamma^2 \int_0^t w^T(\tau) w(\tau) d\tau \\ &\qquad \leq - \ x^T(t) E^T X_{c\ell} x(t) + x^T(0) E^T X_{c\ell} x(0). \end{aligned} \tag{11}$$

If $Ex(0) = 0$, then from (11) we have

$$\int_0^t z^T(\tau) z(\tau) d\tau \leq \gamma^2 \int_0^t w^T(\tau) w(\tau) d\tau,$$

and therefore $\|z\|_{L_2} \leq \gamma \|w\|_{L_2}$. (Q.E.D.)

4. Problem Formulation via Matrix Inequality

In this section, applying Proposition , we formulate our synthesis problem by matrix inequality.

Consider a controller with q-dimensional vector process $x_c(t) \in \mathbf{R}^q$:

$$\begin{cases} \dot{x}_c(t) = K_4 y(t), \ \ x_c(0) = 0 \\ u(t) = K_1 y(t) + K_2 x_c(t) + K_3 \dot{y}(t), \end{cases} \tag{12}$$

where $K_4 \in \mathbf{R}^{q\times r}$ is a preassigned constant matrix which is chosen appropriately according to the structure of the PID controller to be synthesized. Setting $K_P = K_1$, $K_I = K_2K_4$ and $K_D = K_3$ yields the PID controller gains in (4). Furthermore, introducing a vector $x(t)$ defined by

$$x(t) := \begin{bmatrix} x_p^T(t) & x_c^T(t) & \dot{x}_p^T(t) \end{bmatrix}^T, \tag{13}$$

we describe the system which consists of (1) and (12) in the descriptor form:

$$\begin{cases} E\dot{x}(t) = Ax(t) + B_v v(t) + B_w w(t) \\ \tilde{v}(t) = C_v x(t) \\ z(t) = C_z x(t) + D_w w(t), \end{cases} \tag{14}$$

where

$$E := \begin{bmatrix} I_n & 0 & 0 \\ 0 & I_q & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A := A_1 + B_1 K C_1$$

$$A_1 := \begin{bmatrix} 0 & 0 & I_n \\ K_4 C_y & 0 & 0 \\ A_p & 0 & -I_n \end{bmatrix}, \quad B_1 := \begin{bmatrix} 0 \\ 0 \\ B_{pu} \end{bmatrix}$$

$$K := \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix}, \quad C_1 := \begin{bmatrix} C_y & 0 & 0 \\ 0 & I_q & 0 \\ 0 & 0 & C_y \end{bmatrix}$$

$$B_v := \begin{bmatrix} 0 \\ 0 \\ B_{pv} \end{bmatrix}, \quad B_w := \begin{bmatrix} 0 \\ 0 \\ B_{pw} \end{bmatrix}$$

$$C_v := \begin{bmatrix} C_{vp} & 0 & 0 \end{bmatrix}, \quad C_z := A_2 + B_2 K C_1$$

$$A_2 := \begin{bmatrix} C_{zp} & 0 & 0 \end{bmatrix}, \quad B_2 := D_{zu}, \quad D_w := D_{zw}.$$

Proposition proves that if there exist a nonsingular matrix $X_{c\ell}$, a positive-definite diagonal matrix W, a diagonal matrix H_2 ($0 \le H_2 < H_1 \le I_p$), a matrix K and a positive scalar γ satisfying

$$E^T X_{c\ell} = X_{c\ell}^T E \geq 0 \tag{15}$$

$$\begin{aligned}&\Phi(X_{c\ell}, W, \gamma; K, H_2)\\ &:= \begin{bmatrix} \Phi_{11}(X_{c\ell}, W; K, H_2) & \Phi_{12}(X_{c\ell}, W; H_2) & \Phi_{13}(X_{c\ell}) & \Phi_{14}(K) \\ \Phi_{12}^T(X_{c\ell}, W; H_2) & -2W & 0 & 0 \\ \Phi_{13}^T(X_{c\ell}) & 0 & -\gamma^2 I_s & D_w \\ \Phi_{14}^T(K) & 0 & D_w^T & -I_m \end{bmatrix} < 0,\end{aligned} \tag{16}$$

where

$$\Phi_{11}(X_{c\ell}, W; K, H_2) = (A_1 + B_1 K C_1)^T X_{c\ell} + X_{c\ell}^T (A_1 + B_1 K C_1) - \ H(W; H_2)$$

$$\Phi_{12}(X_{c\ell}, W; H_2) = X_{c\ell}^T B_v + C_v^T (H_1 + H_2) W$$

$$\Phi_{13}(X_{c\ell}) = X_{c\ell}^T B_w$$

$$\Phi_{14}(K) = (A_2 + B_2 K C_1)^T,$$

then there exists a controller (12) (or (4)) which achieves the specification that the closed-loop system satisfies the condition $\|z\|_{L_2} \leq \gamma \|w\|_{L_2}$ for $Ex(0) = 0$.

Thus, our problem of determining the controller gain matrix K is formulated as follows:

find K, $X_{c\ell}$, W, H_2 and γ so as to

minimize γ and $\mathrm{tr}(H_2)$

subject to (15) and (16).

This is a BMI problem, and it is difficult to convert it into a sort of convex optimization problems such as LMI problem.

5. Synthesis of PID Controller

In this section, noting that fixing either $\{X_{c\ell}, W\}$ or $\{K, H_2\}$ in (16) yields LMI in the other pair of matrices, we propose an iterative LMI algorithm to solve our problem. The algorithm does not have a proof for global convergence.

In order to solve the problem of finding $X_{c\ell}$, W and γ so as to minimize γ subject to (15) and (16) with fixed K and H_2, we need to calculate K and H_2 in advance.

First, we obtain K by solving the problem of synthesizing a PID controller for a linearized plant, where the linearized plant is obtained by the approximation $\psi(\tilde{v}) \cong \tilde{v}$ (or $H_1 = H_2 = I_p$). Then, the resultant linearized closed-loop system is described as follows:

$$\begin{cases} E\dot{x}(t) = (\tilde{A}_1 + B_1 K C_1)x(t) + B_w w(t) \\ z(t) = (A_2 + B_2 K C_1)x(t) + D_w w(t), \end{cases} \tag{17}$$

where $\tilde{A}_1 = A_1 + A_v$ and $A_v = B_v C_v$.

The controller gain K in the linearized system (17) is denoted by K_ℓ in the sequel. Given a matrix K_ℓ and a positive scalar $\tilde{\gamma}$, the system (17) is admissible and $\|T_{zw}\|_\infty < \tilde{\gamma}$ holds, where $T_{zw}(s)$ is the transfer function matrix from w to z, if and only if there exists a nonsingular matrix Y such that

$$E^T Y = Y^T E \geq 0 \tag{18}$$

$$\begin{aligned} &\Theta(Y, \tilde{\gamma}^2; K_\ell) \\ &:= \begin{bmatrix} (\tilde{A}_1 + B_1 K_\ell C_1)^T Y + Y^T(\tilde{A}_1 + B_1 K_\ell C_1) & (B_w^T Y)^T & (A_2 + B_2 K_\ell C_1)^T \\ B_w^T Y & -\tilde{\gamma}^2 I_s & D_w^T \\ A_2 + B_2 K_\ell C_1 & D_w & -I_m \end{bmatrix} < 0 \end{aligned} \tag{19}$$

hold[10].

Consequently, the problem of synthesizing PID controller under H_∞ criterion, i.e., $\|T_{zw}\|_\infty < \tilde{\gamma}$ is formulated as that of finding matrices Y, K_ℓ and a positive scalar $\tilde{\gamma}$ satisfying the conditions (18) and (19). Note that (19) is BMI in Y and K_ℓ.

Noting that fixing either Y or K_ℓ in (19) yields the LMI condition in the other matrix, we minimize $\tilde{\gamma}^2$ locally by the iterative LMI algorithm for Y and K_ℓ. Here, since the algorithm requires an initial value of PID controller gain, first, the initial value K_{int} is set by K_ℓ such that the matrix $\tilde{A}_1 + B_1 K_\ell C_1$ is stable.

Procedure for Obtaining K_ℓ

Step 1: Set $K_{\mathrm{int}} \leftarrow K_\ell$, find Y and $\tilde{\gamma}^2$ so as to minimize $\tilde{\gamma}^2$ subject to $E^T Y = Y^T E \geq 0$ and $\Theta(Y, \tilde{\gamma}^2; K_{\mathrm{int}}) < 0$ in (18) and (19) ($i \leftarrow 1, Y_i \leftarrow Y$ and $\tilde{\gamma}_i^2 \leftarrow \tilde{\gamma}^2$).

Step 2: With the obtained Y_i and $\tilde{\gamma}_i^2$, find K_i subject to $\Theta(Y_i, \hat{\gamma}_i^2; K_i) < 0$ in (19).

Step 3: Again, with K_i in Step 2, find Y and $\tilde{\gamma}^2$ so as to minimize $\tilde{\gamma}^2$ subject to $E^T Y = Y^T E \geq 0$ and $\Theta(Y, \tilde{\gamma}^2; K_i) < 0$ in (18) and (19) ($Y_{i+1} \leftarrow Y$ and $\tilde{\gamma}_{i+1}^2 \leftarrow \tilde{\gamma}^2$).

Step 4: If $|\tilde{\gamma}_{i+1} - \tilde{\gamma}_i| < \varepsilon$, where ε is the prescribed tolerance, stop. Otherwise, let $i \leftarrow i + 1$ and go back to Step 2.

Using K_ℓ obtained by the above procedure, we state an iterative LMI algorithm for synthesis of PID controller (12) for a nonlinear plant (1) with (2) as follows:

An Iterative LMI Algorithm

Step 1: Choose a scalar positive constant ε and matrices H_1 and H_2, and then set $K^* \leftarrow K_\ell$ and $H_2^* \leftarrow H_2$.

Step 2: Find $X_{c\ell}, W(> 0)$ and γ so as to minimize γ subject to $E^T X_{c\ell} = X_{c\ell}^T E \geq 0$ and $\Phi(X_{c\ell}, W, \gamma; K^*, H_2^*) < 0$ in (15) and (16). Set $X_{c\ell}^* \leftarrow X_{c\ell}, W^* \leftarrow W, \gamma^* \leftarrow \gamma$.

Step 3: Find K and H_2 so as to minimize tr(H_2) subject to $\Phi(X_{c\ell}^*, W^*, \gamma^*; K, H_2) < 0$ in (16).

Step 4: If tr$(H_2^* - H_2) < \delta$, then stop the algorithm. Otherwise, set $K^* \leftarrow K_\ell$, $H_2^* \leftarrow H_2$, and go to Step 2.

6. A Numerical Example

In this section, we apply the approach proposed in this paper to the problem[12] of synthesizing a dynamic controller to balance a ball on the periphery of a wheel as shown in Fig. 2, where θ_1 is the angle between the center of the ball and the vertical axis, θ_2 is the wheel angular position, and τ and w are the control torque exerted on the wheel and the disturbance added to the wheel, respectively. I_w is the inertia of the wheel, m_b is the mass of the ball, and r_b, r_w are the radii of the ball and the wheel, respectively. g is the gravitational acceleration. The physical parameters are listed in Table 1, in which the numerical values are the same as in M. Ho and J. Lu[12].

Table 1. Physical parameters of ball-on-wheel system.

r_b	radius of the ball	0.0125 [m]
r_w	radius of the wheel	0.121 [m]
I_w	inertia of the wheel	9.938×10^{-3} [kgm^2]
m_b	mass of the ball	0.065 [kg]
R_a	motor armature resistance	1.6 [Ω]
K_m	motor constant	0.10352 [Nm/A]
g	gravitational acceleration	9.8 [N/m^2]

In this paper, we assume that the coefficient of friction is sufficiently large and therefore the ball rolls on the wheel without slipping. Then, the equations of motion of the

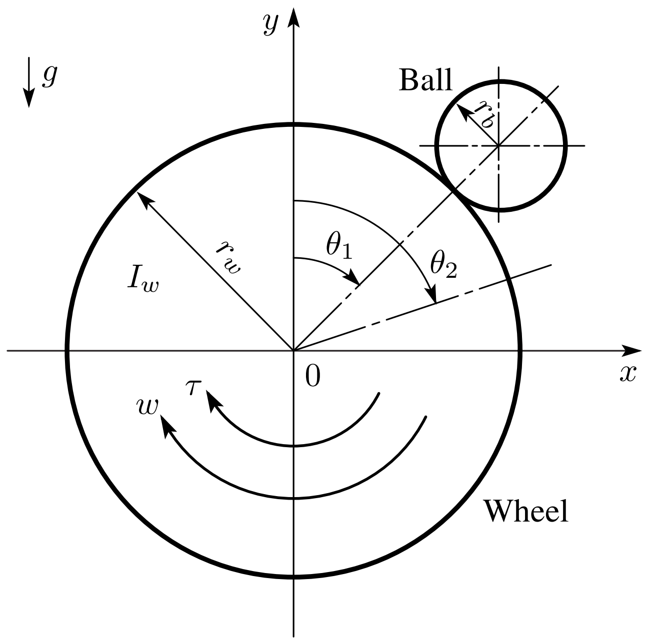

Fig. 2. Ball-on-wheel system.

system can be written as follows:

$$\begin{cases} -\dfrac{2}{5}m_b r_w \left(r_w + r_b\right)\ddot{\theta}_1(t) + \left(I_w + \dfrac{2}{5}m_b r_w^2\right)\ddot{\theta}_2(t) = \tau(t) + w(t) \\ -7\left(r_b + r_w\right)\ddot{\theta}_1(t) + 2r_w\ddot{\theta}_2(t) + 5g\sin\theta_1(t) = 0. \end{cases} \tag{20}$$

A voltage signal $u(t)$ is generated according to the desired control law and it is supplied to an amplifier which drives a permanent magnet DC motor to control the wheel. The relation between the control torque $\tau(t)$ and the control voltage $u(t)$ is given by

$$\tau(t) = \frac{K_m}{R_a}u(t) - \frac{K_m^2}{R_a}\dot{\theta}_2(t), \tag{21}$$

where R_a is the motor armature resistance and K_m is the motor constant.

Here, note that to maintain the ball on the wheel the centripetal force must be larger than the centrifugal force:

$$g\ \cos\theta_1 > (r_b + r_w)\dot{\theta}_1^2. \tag{22}$$

We define the state variables as

$$x_p(t) := \left[\begin{array}{cccc} \theta_1(t) & \dot{\theta}_1(t) & \theta_2(t) & \dot{\theta}_2(t) \end{array}\right]^T,$$

and set

$$y(t) = z(t) = c_1\theta_1(t) + c_3\theta_2(t),$$

where the sensor gains are $c_1 = 10[\mathrm{V/rad}]$ and $c_3 = -1[\mathrm{V/rad}]$. Then, from (20) and (21), we see that the ball-on-wheel system is described by

$$v = \phi(\tilde{v}) := \sin\tilde{v}, \tag{23}$$

and (1) with the following coefficient matrices:

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \bar{a} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \bar{d} \end{bmatrix}, \quad B_{pv} = \begin{bmatrix} 0 \\ \bar{b} \\ 0 \\ \bar{e} \end{bmatrix}, \quad B_{pu} = \begin{bmatrix} 0 \\ \bar{c} \\ 0 \\ \bar{f} \end{bmatrix}$$

$$B_{pw} = \begin{bmatrix} 0 & \bar{g}_1 & 0 & \bar{g}_2 \end{bmatrix}^T$$

$$C_{vp} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad C_{yp} = C_{zp} = \begin{bmatrix} c_1 & 0 & c_3 & 0 \end{bmatrix}$$

$$D_{zu} = 0, \quad D_{zw} = 0, \quad D_{yw} = 0$$

$$\alpha = \left(2m_b r_w^2 + 7I_w\right)(r_b + r_w), \quad \bar{a} = -2r_w K_m^2/R_a\alpha$$

$$\bar{b} = 5g\left(I_w + \frac{2}{5}m_b r_w^2\right)/\alpha, \quad \bar{c} = 2r_w K_m/R_a\alpha$$

$$\bar{d} = -7\left(r_b + r_w\right)K_m^2/R_a\alpha, \quad \bar{e} = 2m_b g r_w\left(r_b + r_w\right)/\alpha$$

$$\bar{f} = 7(r_b + r_w)K_m/R_a\alpha, \quad \bar{g}_1 = 2r_w/\alpha$$

$$\bar{g}_2 = 7(r_b + r_w)/\alpha.$$

The relation between the function $\phi(\tilde{v})$ and the sector is illustrated in Fig. 3. This figure shows that $|\tilde{v}| \leq M$ implies $\phi \in \Delta$.

The approximation $\phi(\tilde{v}) \cong \tilde{v}$ (or $\sin\theta_1 \cong \theta_1$) yields a linearized closed-loop system described by (17). Then, setting

$$K_4 = 1, \ K_{\mathrm{int}} = \begin{bmatrix} -30 & -350 & -2 \end{bmatrix}, \ \varepsilon = 0.001,$$

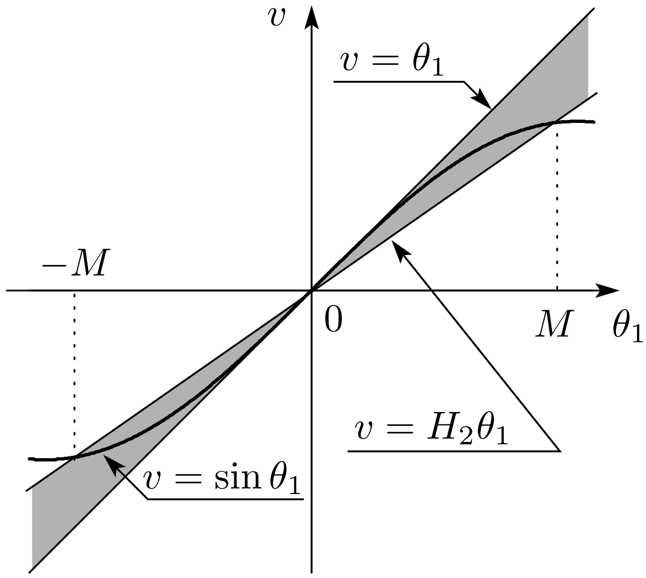

Fig. 3. Relation between ϕ and sector.

and using the procedure mentioned in the previous section, after 40 iterations in the iterative LMI algorithm, we obtain

$$K_\ell = \begin{bmatrix} K_P & K_I & K_D \end{bmatrix} = \begin{bmatrix} -25.2420 & -96.6606 & -0.9425 \end{bmatrix}$$

$$\tilde{\gamma} = 5.4448.$$

We choose a scalar δ and an initial value of $H_2 (=: H_2^o)$ as follows:

$$\delta = 0.0001, \quad H_2^o = 0.99995.$$

After 16 iterations in the iterative LMI algorithm given in Section 5, we obtain

$$K = \begin{bmatrix} K_P & K_I & K_D \end{bmatrix} = \begin{bmatrix} -35.0384 & -379.6858 & -4.1971 \end{bmatrix}$$

$$W = 191.2726, \quad \gamma = 1.2778, \quad H_2 = 0.8588,$$

$$M = 53.9112\,[\mathrm{deg}](= 0.9409\,[\mathrm{rad}]).$$

Figures 4 and 5 show the value of performance index γ and the slope H_2 for each step, respectively.

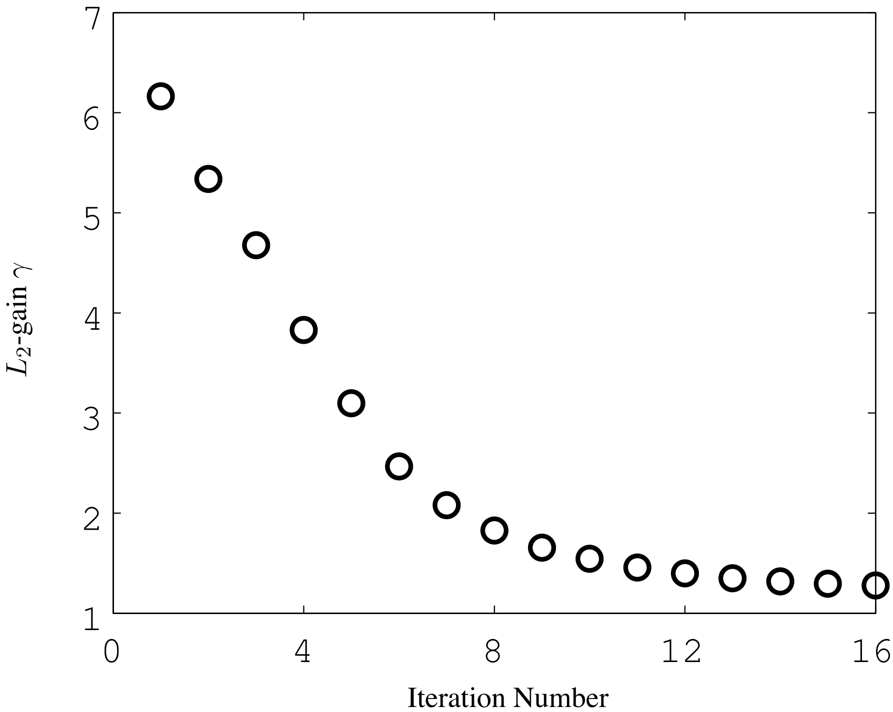

Fig. 4. L_2-gain γ.

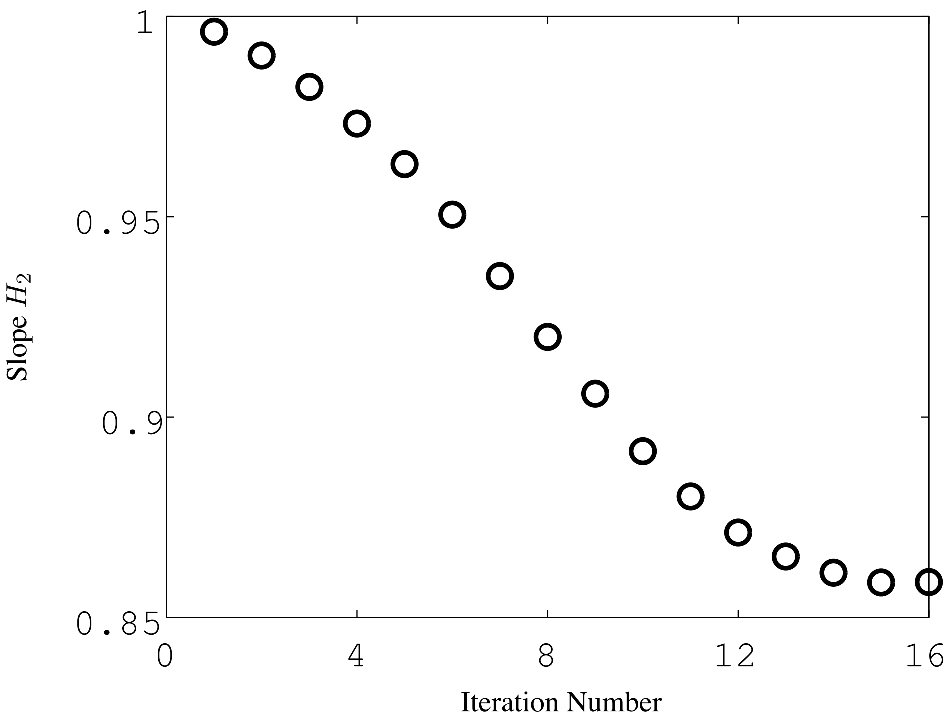

Fig. 5. The slope H_2.

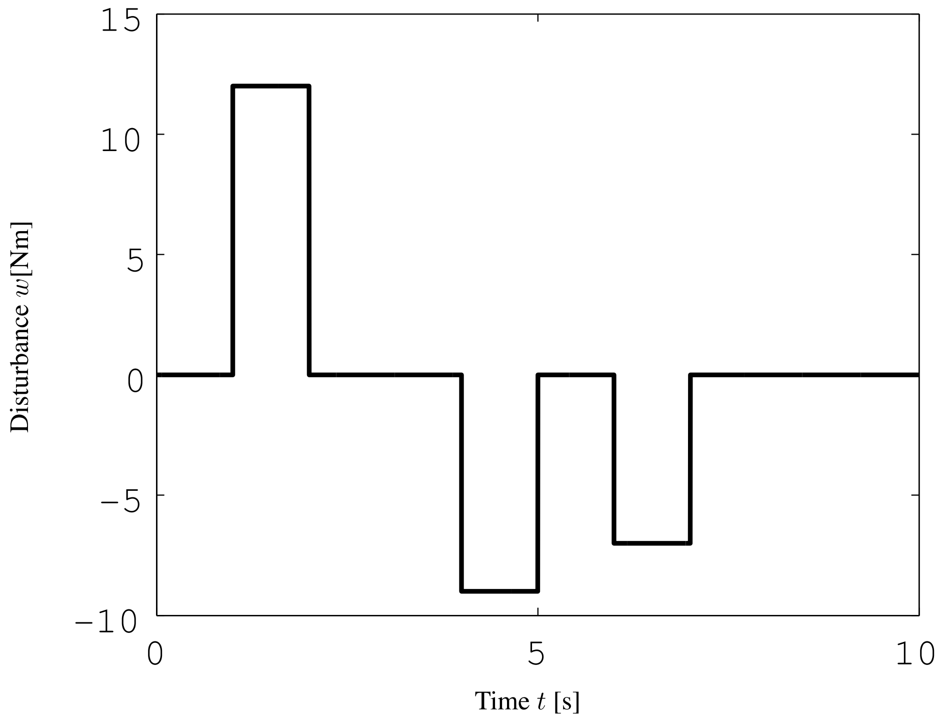

Fig. 6. Disturbance w.

Disturbance $w(t)$ added to the wheel is shown in Fig. 6. Behavior of $\theta_1(t)$, $\theta_2(t)$ and $\tau(t)$ is depicted in Figs. 7 to 9. We see from Fig. 10 that the constraint (22) is satisfied.

7. Conclusion

A method of synthesizing multivariable PID controllers for systems with sector-bounded nonlinearity has been proposed. In order to expand the region of the sector and minimize the L_2-gain, an iterative LMI algorithm has been presented. Although the algorithm does not have a proof for global convergence, it has a practical use. The authors have applied the algorithm to design of the PID controller of a ball-on-wheel system, and confirmed the effectiveness numerically.

Acknowledgment

The authors would like to appreciate Mr. Masayuki Maeda, graduate student of Kyoto Institrute of Technology for his assistance in simulation work.

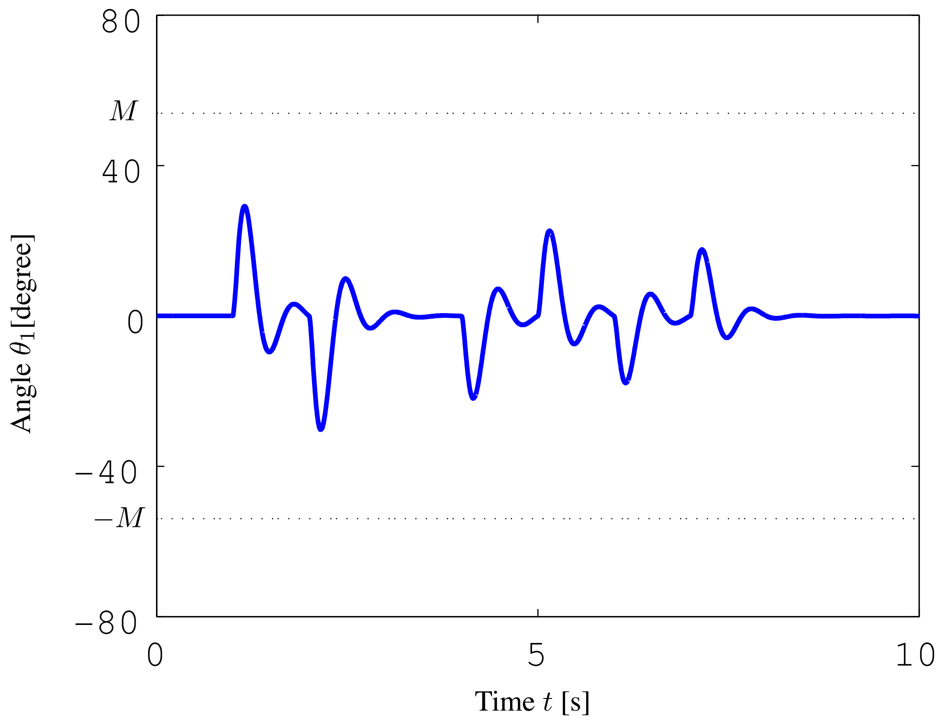

Fig. 7. The angle θ_1.

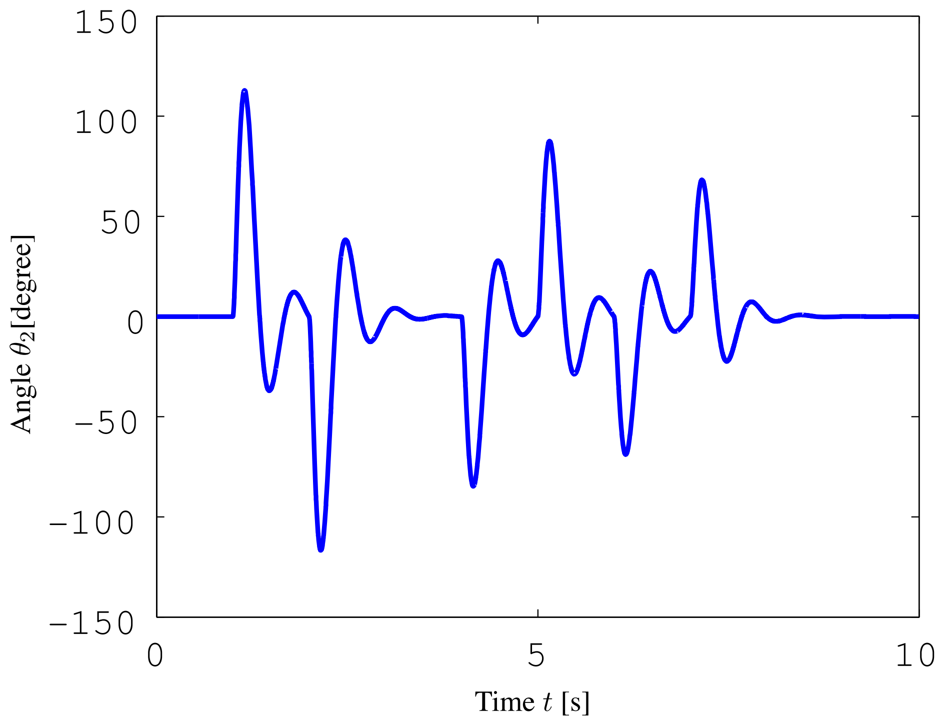

Fig. 8. The angle θ_2.

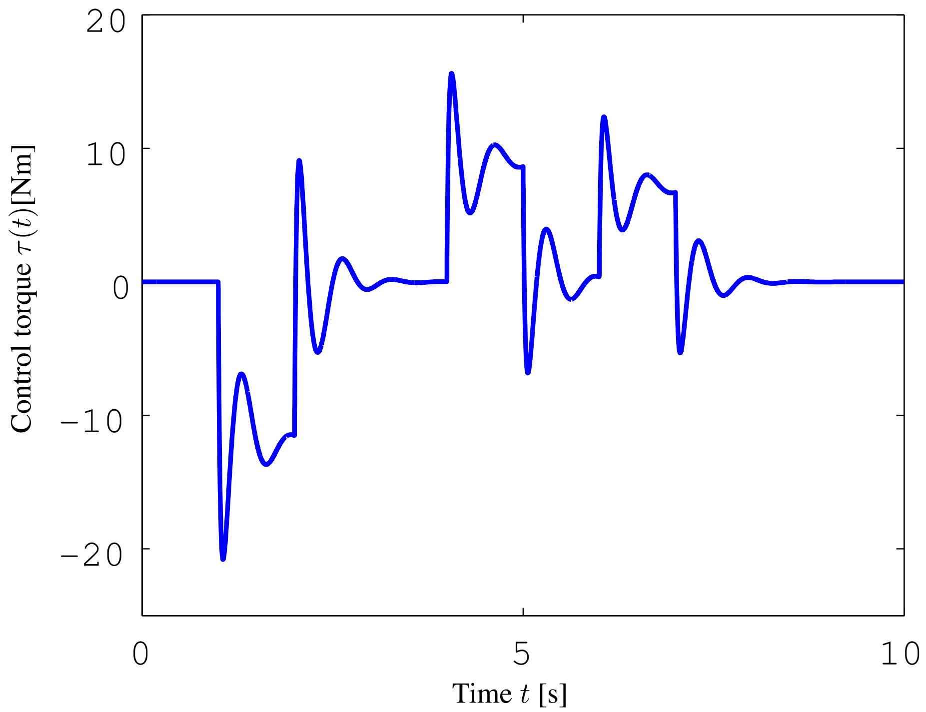

Fig. 9. The torque τ.

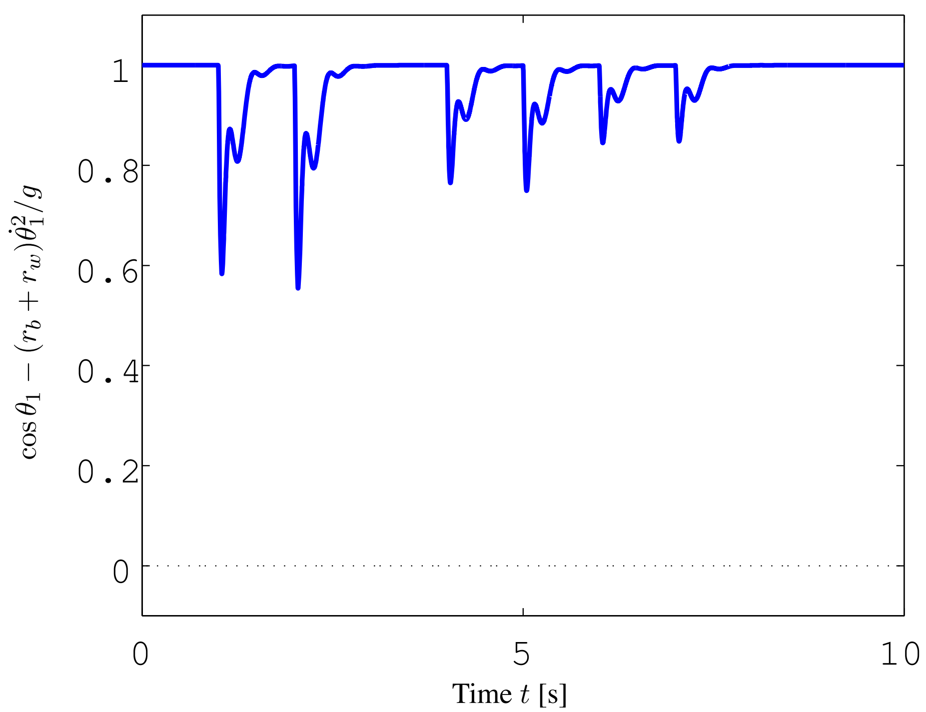

Fig. 10. The constraint $\cos\theta_1 - (r_b + r_w)\dot{\theta}_1^2/g > 0$ in (22).

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非線形系に対する多変数 PID コントローラの設計

本論文では，非線形性を有する制御対象に対する多変数比例・積分・微分 (PID) コントローラの設計手法を検討する．はじめに，セクタ有界非線形性を有する制御対象と多変数 PID コントローラの数学モデルを記述する．つぎに，その閉ループ制御系をデスクリプタ形式で表現する．セクタ有界非線形性を有するデスクリプタ系の L_2-運転性能を検証するための十分条件を導入する．その条件に基づいて，多変数 PID コントローラ設計問題を双線形行列不等式 (BMI) 問題として定式化する．本論文では，考慮される非線形性の領域を拡げ，L_2-ゲインを最小にするように PID コントローラを設計するための繰り返しアルゴリズムを提案する．最後に，提案手法の有効性を検証するために，ball-on-wheel 系に対して PID コントローラを設計する数値例を示す．

キーワード: 多変数 *PID* コントローラ; セクタ有界非線形性; デスクリプタ系; 行列不等式; L_2-運転性能.