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Low Grazing Scattering from Sinusoidal Neumann Surface with Finite Extent: Undersampling Approximation

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SUMMARY A transverse magnetic (TM) plane wave is diffracted by a periodic surface into discrete directions. However, only the reflection and no diffraction take place when the angle of incidence becomes a low grazing limit. On the other hand, the scattering occurs even at such a limit, if the periodic surface is finite in extent. To solve such contradiction, this paper deals with the scattering from a perfectly conductive sinusoidal surface with finite extent. By the undersampling approximation introduced previously, the total scattering cross section is numerically calculated against the angle of incidence for several corrugation widths up to more than 10^4 times of wavelength. It is then found that the total scattering cross section is linearly proportional to the corrugation width in general. But an exception takes place at a low grazing limit of incidence, where the total scattering cross section increases almost proportional to the square root of the corrugation width. This suggests that, when the corrugation width goes to infinity, the total scattering cross section diverges and the total scattering cross section per unit surface vanishes at a low grazing limit of incidence. Then, it is concluded that, at a low grazing limit of incidence, no diffraction takes place by a periodic surface with infinite extent and the scattering occurs from a periodic surface with finite extent.
key words: *TM wave scattering, finite periodic surface, total scattering cross section, total scattering cross section per unit surface, diffraction cross section, low grazing angle of incidence*

1. Introduction

This paper deals with the scattering of a TM plane wave from a perfectly conductive sinusoidal surface with a finite corrugation width W (See Fig. 1). We study the scattering at a low grazing limit of incidence (LGLI) with $\theta_i \to 0$.

A ground based high frequency (HF) radar detects the backscattering from a rough sea surface, tens of kilometers beyond the horizon [1]. In this remote sensing problem, a rough sea surface is physically regarded as a large diffraction grating with roughness much smaller than the wavelength. Then, the scattering and diffraction by such a grating at LGLI are subjects of great interest.

The diffraction by a rough periodic surface becomes singular at LGLI. No diffraction takes place and only the reflection occurs at LGLI. For a perfectly conductive sinusoidal grating, Ikuno and Yasuura [2] obtained rigorous numerical results such that the 0th order diffraction efficiency becomes unity and any other order diffraction efficiencies vanish at LGLI. Such behavior of diffraction efficiencies was found numerically for a plane grating with reflector [3] and for a periodic array of dielectric cylinders [4]. By use of the grazing perturbation method, however, Charnotskii [5] found analytically that non-zero order diffraction amplitudes vanish and the complex reflection coefficient becomes −1 at LGLI in the case of a slightly rough periodic Neumann surface. Such behavior of the diffraction amplitudes was verified for any periodic Neumann surface with small roughness and gentle slope by use of the modified diffraction amplitude [6].

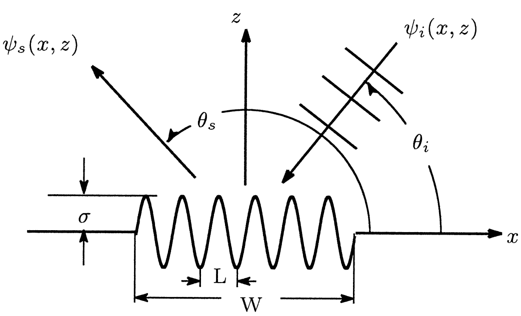

Fig. 1 Scattering of a plane wave from a sinusoidal surface with finite extent. L is the period, σ is the surface roughness, and W is the corrugation width. θ_i and θ_s are the angle of incidence and a scattering angle, respectively.

On the other hand, it is analytically predicted that the scattering may take place even at LGLI, if the rough surface is finite in extent [7]. This prediction is verified numerically for a finite sinusoidal surface [8], [9] and for a finite periodic array of rectangular grooves [10].

Obviously, we have a wide gap between the diffraction theory and the scattering theory. The former says only the reflection takes place but the latter insists the scattering occurs at LGLI. However, it seems that this contradiction has not been discussed extensively. When the corrugation width W goes to infinity, a periodic surface with finite extent approaches to a perfectly periodic surface. This fact suggests that the asymptotic behavior of the scattering for a sufficiently large W may give a solution to the contradiction. Taking this idea, we numerically calculated the total scattering cross section p_c of a finite periodic array of rectangular grooves for several different numbers of grooves [10].

This paper discusses another example, which is the scattering of a TM plane wave from a sinusoidal surface with finite extent (See Fig. 1). By the undersampling approximation [9], we numerically calculate the total scattering cross section p_c for several corrugation widths up to more than $10^4\lambda$, λ being wavelength. For a sinusoidal surface with infinite extent (sinusoidal grating), we also calcu-

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late the diffraction cross section $p_c^{(g)}$. Then, we find several properties as follows. (A) For any angle of incidence, p_c increases when W becomes large. (B) When the angle of incidence θ_i is apparently different from any one of the critical angles of incidence, p_c increases linearly proportional to W and the total scattering cross section per unit surface p_c/W is almost equal to $p_c^{(g)}$ the diffraction cross section. (C) When θ_i is critical but is not grazing, p_c/W depends on W and becomes close to $p_c^{(g)}$ when W becomes large. These properties are much similar to those in the case of a finite periodic array of grooves [10]. Furthermore, we find a new property such that (D) at LGLI p_c increases almost proportional to $\sqrt{W/\lambda}$ and p_c/W decreases when W becomes large.

In this paper, a time variation $e^{-i\omega t}$ is assumed and suppressed.

2. Formulation

Let us consider the scattering of a TM plane wave from a perfectly conductive sinusoidal surface with finite extent. We write the surface corrugation as

$$z = f(x) = \sigma u(x|W)\sin(k_L x), \quad k_L = \frac{2\pi}{L}, \tag{1}$$

$$u(x|W) = u^2(x|W) = \begin{cases} 1, & |x| \le W/2 \\ 0, & |x| > W/2 \end{cases}, \tag{2}$$

where L is the period, k_L is the spatial angular frequency of the period L, σ is the surface roughness, $u(x|W)$ is a rectangular pulse and W is a corrugation width. We implicitly assume W is an integer multiple of the period L to make $f(x)$ continuous at $x = \pm W/2$. In what follows, we only consider a case with $\sigma \ll \lambda$, λ being wavelength. We denote the y component of the magnetic field by $\psi(x,z)$, which satisfies the Helmholtz equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right]\psi(x,z) = 0, \tag{3}$$

in the region $z > f(x)$ and the Neumann condition on the surface (1)

$$\left[\frac{\partial}{\partial z} - \frac{df}{dx}\frac{\partial}{\partial x}\right]\psi(x,z)\bigg|_{z=f(x)} = 0. \tag{4}$$

Here, $k = 2\pi/\lambda$ is wavenumber.

Since the surface is flat for $|x| > W/2$, we write the wave field $\psi(x,z)$ as

$$\psi(x,z) = e^{-ipx}e^{-i\beta(p)z} + e^{-ipx}e^{i\beta(p)z} + \psi_s(x,z), \tag{5}$$

where the first term on the right-hand side is the incident plane wave, the second term is the specularly reflected wave and $\psi_s(x,z)$ is the scattered wave due to the surface corrugation. Here, p is given by the angle of incidence θ_i as (See Fig. 1)

$$p = k\cos\theta_i, \tag{6}$$

and $\beta(p)$ is a complex function of p,

$$\beta(p) = \sqrt{k^2 - p^2},$$
$$Re[\beta(p)] \ge 0, \quad Im[\beta(p)] \ge 0, \tag{7}$$

where Re and Im denote real and imaginary parts, respectively.

In the far region, $\psi_s(x,z)$ becomes a cylindrical wave satisfying the Sommerfeld radiation condition. We write an approximate expression of $\psi_s(x,z)$ as

$$\psi_s(x,z) = \int_{-k_B}^{k_B} \frac{A_\beta(s)}{\beta(p+s)} e^{-i(p+s)x + i\beta(p+s)z} ds, \tag{8}$$

which is made up of up-going plane waves and evanescent waves. Here, k_B is a truncated band width, and $A_\beta(s)$ is the angular spectrum representing the amplitude of the partial wave scattered into $\theta_s = \Theta(p+s)$ direction. Here, $\Theta(p+s)$ is defined by a geometric relation,

$$\Theta(p+s) = \arccos[-(p+s)/k]. \tag{9}$$

If we put $s = mk_L$, $(m = 0, \pm1, \pm2, \cdots)$, this becomes a grating formula [11] for a perfectly periodic surface,

$$\Theta_m = \Theta(p + mk_L) = \arccos[-(p + mk_L)/k], \tag{10}$$

where Θ_m is the mth order diffraction angle.

In the scattering theory, the energy conservation appears as the optical theorem (the forward scattering theorem), stating that the total scattering cross section of a non-absorbing target with finite extent is proportional to the imaginary part of the forward scattering amplitude [12]. In a finite periodic case, the optical theorem means that the total scattering cross section is proportional to the real part of the specularly scattering amplitude and is written as [8],

$$p_c = p_{inc}, \tag{11}$$

$$p_c = -\frac{4\pi}{k} Re[A_\beta(0)], \tag{12}$$

$$p_{inc} = \frac{W}{2\pi}\int_0^\pi \sigma_s(\theta_s|\theta_i)d\theta_s, \tag{13}$$

$$\sigma_s(\theta_s|\theta_i) = \frac{4\pi^2}{kW}|A_\beta(-k\cos\theta_s - k\cos\theta_i)|^2. \tag{14}$$

Here, $\sigma_s(\theta_s|\theta_i)$ is the differential scattering cross section per unit surface and has no dimension. The optical theorem (11) states that the total scattering cross section p_{inc} is equal to p_c the loss of the amplitude of the partial wave scattered into the specular reflection direction†. Because of (11), however, we will call p_c the total scattering cross section.

The optical theorem may be useful to estimate the accuracy of an approximate solution (8). We define the error E_{rr} with respect to the optical theorem as,

$$E_{rr} = \left|\frac{p_c - p_{inc}}{p_c}\right|, \tag{15}$$

†'$Re[A_\beta(0)]$ is obtained from the amplitude and phase of the scattered wave, which can be easily measured in radio frequencies. We also note that p_{inc} was called the total power of scattering in [8] but is called the total scattering cross section in this paper.

which will be calculated below.

In what follows, we will determine $A_\beta(s)$ by the undersampling approximation to calculate the total scattering cross section p_c, the total scattering cross section per unit surface p_c/W, the differential scattering cross section $\sigma_s(\theta_s|\theta_i)$ and E_{rr} the error with respect to the optical theorem.

2.1 Undersampling Approximation

Rigorous numerical methods, such as the integral equation method [13], Yasuura's method [14], [15] and the spectral formalism [8], commonly reduce the scattering problem to solving a linear equation system. The system is about $[8\,W/\lambda] \times [8\,W/\lambda]$ in size and hence it becomes impractical to solve a case with $W/\lambda > 10^3$. On the other hand, the undersampling approximation starts with a physical picture such that, when W is sufficiently large, the finite periodic surface works as a diffraction grating. This means that the scattered wave in the far region is well approximated by a sum of beams diffracted into directions given by the grating formula (10). The undersampling approximation reduces the matrix size to $[FL/\lambda + 1] \times [FL/\lambda + 1]$, where F is an empirical factor between 4 and 8, and hence enables us to obtain an approximate solution for a wide case with $W/\lambda > 10^4$.

In the undersampling approximation [9], $A_\beta(s)$ is approximated by

$$A_\beta(s) = \frac{1}{2\pi} \sum_{m=-N_Q}^{N_Q} Q_m U(s - mk_L|W), \tag{16}$$

$$U(s|W) = \int_{-\infty}^{\infty} u(x|W) e^{isx} dx = \frac{\sin(sW/2)}{(s/2)}, \tag{17}$$

where $[Q_m]$ is the vector to be determined and N_Q is a truncation number. Physically, $U(s - mk_L|W)$ represents the beam pattern of the mth order diffraction beam, the mainlobe of which is scattered into the $\theta_s = \Theta_m$ direction given by (10). Let us obtain the 3 dB beam width of the mth order diffraction beam for a sufficiently large W. Since $U^2(s|W)/W^2 = [\sin(sW/2)/(sW/2)]^2 \approx 1/2$ at $s = \pm 2.78/W$, the (complex) beam width $\Delta\Theta_s$ is roughly estimated by a geometric relation as

$$\begin{aligned}\Delta\Theta_s &= \Theta\left(p + mk_L + \frac{2.78}{W}\right) - \Theta\left(p + mk_L - \frac{2.78}{W}\right) \\ &\approx \begin{cases} 0.885\lambda/(W \sin\Theta_m), & \sin\Theta_m \neq 0 \\ 0.940(1-i)\sqrt{\lambda/W}, & \sin\Theta_m = 0 \end{cases}.\end{aligned} \tag{18}$$

When $\sin\Theta_m$ is positive real, the mth order diffraction beam is propagating and has a real beam width $\Delta\Theta_s$ proportional to λ/W. If $\sin\Theta_m$ is pure imaginary, such the beam becomes evanescent. When $\sin\Theta_m = 0$, the mth order diffraction beam is scattered into a grazing direction. Its mainlobe is made up with evanescent waves and propagating waves, and hence the beam width $\Delta\Theta_s$ becomes complex. Note that the diffraction beam scattered into a grazing direction has a much wide beam width proportional to $\sqrt{\lambda/W}$. This phenomenon was first found out by a numerical analysis [8].

Using the boundary condition (4) and (8), we may obtain a linear equation system for $[Q_n]$ as

$$\sum_{n=-N_Q}^{N_Q} D_{ln}(p) Q_n = E_l(p), \qquad (l = 0, \pm 1, \pm 2, \cdots, \pm N_Q), \tag{19}$$

where

$$D_{ln}(p) = \sum_{m=-\infty}^{\infty} \int_{-k_B}^{k_B} \frac{U(s - nk_L|W)}{2\pi\, W\beta(p+s)} \times C_m(p+s, \beta(p+s)) U(s + (m-l)k_L|W) ds, \tag{20}$$

$$E_l(p) = -\left[C_l(p, -\beta(p)) + C_l(p, \beta(p))\right], \tag{21}$$

$$C_m(\alpha, \beta) = \beta J_{-m}(\sigma\beta) + \frac{\sigma\alpha k_L}{2}\left[J_{1-m}(\sigma\beta) + J_{-1-m}(\sigma\beta)\right], \tag{22}$$

and $J_m(\cdot)$ is the Bessel function. We will numerically calculate $D_{ln}(p)$ in (20) and then we solve (19) to obtain $[Q_n]$. However, we note that numerical integrations to obtain $[D_{ln}(p)]$ take much computation time when W becomes large.

3. Relation with Periodic Grating

In the limit $W \to \infty$, our surface (1) becomes perfectly periodic and hence the scattered wave $\psi_s(x, z)$ is physically expected to converge to the diffracted wave by the perfectly periodic surface. Mathematically, however, such convergence is doubtful[†]. However, we expect that p_c/W the scattering cross section per unit surface converges to $p_c^{(g)}$ the diffraction cross section of the periodic surface.

Let us consider the diffraction by the perfectly periodic surface. As is well known, the diffracted wave has the Floquet form in a perfectly periodic case. According to reference [16], we write

$$\psi(x, z) = e^{-ipx} e^{-i\beta(p)z} + e^{-ipx} e^{i\beta(p)z} + \sum_{m=-N_{FM}}^{N_{FM}} A_m e^{-i(p+mk_L)x} e^{i\beta(p+mk_L)z}, \tag{23}$$

[†]The convergence from (5) to (23) is still an open question. However, we point out two facts. First, (26) and (31) imply that $\lim_{W\to\infty} \lim_{\theta_i\to 0}[p_c/W - p_c^{(g)}] = 0$ and $\lim_{W\to\infty} \lim_{\theta_i\to 0}[p_c - Wp_c^{(g)}] = \infty$. Second, Fig. 9 suggests that $\sigma_s(\theta_s|\theta_i)$ at LGLI converges to a non-continuous function of θ_s when $W \to \infty$. The sidelobes of $\sigma_s(\theta_s|\theta_i)$ decrease proportional to W and vanish at the limit $W \to \infty$. The beam width $\Delta\Theta_s$ for any diffraction beam goes to zero at such the limit. Furthermore. Figure 9(A) suggests that $\sigma_s(\theta_s|\theta_i)$ takes a positive value at $\theta_s = \Theta_m$ even when $W \to \infty$. Therefore, $\sigma_s(\theta_s|\theta_i)$ converges to a non-continuous function of θ_s, that is, $\sigma_s(\theta_s|\theta_i) = 0$ for any $\theta_s \neq \Theta_m$ and $\sigma_s(\theta_s|\theta_i) > 0$ if $\theta_s = \Theta_m$. Thus, further discussions are needed to solve the convergence problem.

where N_{FM} is a truncation number of Floquet modes, the second term in the right hand side is the reflected wave by a flat surface, and $(A_m + \delta_{m0})$ is the diffraction amplitude of the mth order Floquet mode. Note that $(A_0 + 1)$ is the complex reflection coefficient.

Let us discuss the singular behavior of the diffraction at LGLI. In terms of the optical theorem, we define the total scattering cross section per unit surface $p_c^{(g)}$ as

$$p_c^{(g)} = -2\frac{\beta(p)}{k}Re[A_0] = \sum_{m=-\infty}^{\infty} \frac{Re[\beta(p+mk_L)]}{k}|A_m|^2. \quad (24)$$

However, we simply call $p_c^{(g)}$ the diffraction cross section to distinguish from p_c/W for the finite periodic case.

It is known [16] that an inequality $-2 \le Re[A_0] \le 0$ holds when $\beta(p) > 0$. By this and (24), we then obtain another inequality on the diffraction cross section,

$$0 \le p_c^{(g)} \le 4\sin\theta_i, \quad (25)$$

which will be seen below. If $p_c^{(g)} = 0$ at LGLI, an amplitude A_m $(m \ne 0)$ of a propagating Floquet mode with $Re[\beta(p + mk_L)] > 0$ must vanish by (24). Thus, if $p_c^{(g)} = 0$ at LGLI, no diffraction takes place but only the reflection could occur, which means the diffraction becomes singular at LGLI.

However, it holds that, in the case of a perfectly periodic Neumann surface with small roughness, A_0 becomes -2 (the complex reflection coefficient $A_0 + 1$ becomes -1) and any other diffraction amplitudes A_m, $(m \ne 0)$, vanish at LGLI [5], [6]. Therefore, we obtain from (24),

$$p_c^{(g)} = 4\sin\theta_i \to 0, \quad (\theta_i \to 0), \quad (26)$$

which is illustrated below.

Physically, both p_c/W and $p_c^{(g)}$ are the scattering cross section per unit surface. Therefore, we expect that p_c/W converges to $p_c^{(g)}$ when $W \to \infty$,

$$\frac{p_c}{W} = p_c^{(g)}, \quad (W \to \infty). \quad (27)$$

We will see that (27) holds with good accuracy even when W is finite and sufficiently large. We note that (27) gives a solution to our contradiction such that no diffraction takes place but scattering may occur at LGLI.

4. Numerical Example

For numerical calculations, we put

$$L = 1.75\lambda, \quad \sigma = 0.2\lambda, \quad (28)$$

$$N_Q = N_{FM} = 6, \quad k_B = (N_Q + 1/2)k_L. \quad (29)$$

Since $L/\lambda = 1.75$, the critical angles of incidence are $\theta_i = 0°$, 44.41530°, 64.62306° and 81.78678°, at which one of the diffraction beams is scattered into a grazing direction and then re-scattered again and again by the surface corrugation. Such multiple scattering could give serious effects to the scattering properties. In the perfectly periodic case, such

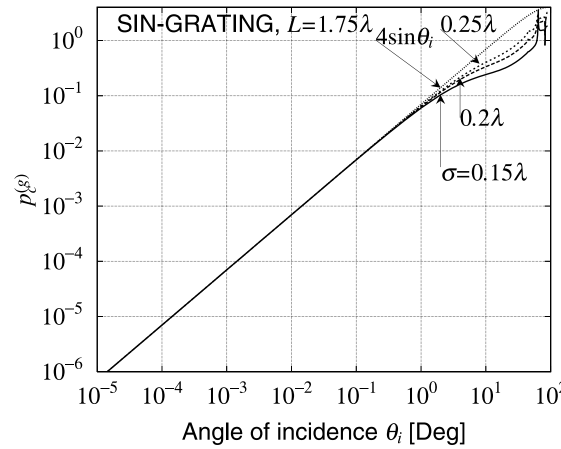

Fig. 2 Diffraction cross section $p_c^{(g)}$ against θ_i the angle of incidence. $L = 1.75\lambda$, $\sigma/\lambda = 0.15$, 0.20 and 0.25.

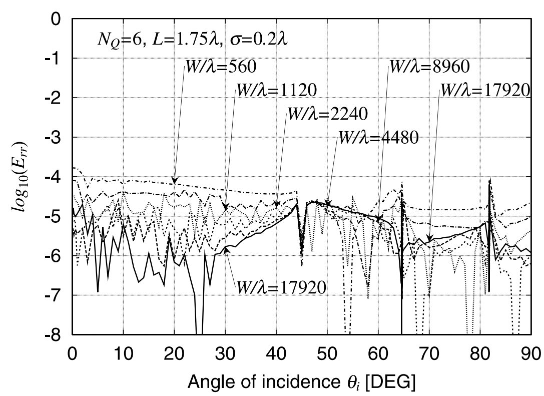

Fig. 3 The error E_{rr} with respect to the optical theorem. $L = 1.75\lambda$, $\sigma = 0.2\lambda$.

effect is widely known as Wood's anomaly. However, multiple scattering effects are not clear in the case of a periodic surface with finite extent.

We first consider the perfectly periodic case. By the Rayleigh method [11], we determine A_m in (23). We then calculate $p_c^{(g)}$ against θ_i the angle of incidence in Fig. 2, where we display $p_c^{(g)}$ for low angles of incidence by a logarithmic graph. We see that the inequality (25) holds for any θ_i. We also see that, when $\theta_i < 0.1°$, $p_c^{(g)}$ becomes almost independent of σ and is almost equal to $4\sin\theta_i$, as is expected by (26).

Next, let us discuss the scattering from a finite sinusoidal surface. We solved numerically (19) to obtain $[Q_n]$ for θ_i from 0.00001° to 90° and for $W/\lambda = 520$, 1120, 2240, 4480, 8960 and 17920, where the error E_{rr} with respect to the optical theorem (15) is less than 1.66×10^{-4} as is illustrated in Fig. 3. However, the error becomes less than 1.13×10^{-4} when $\sigma = 0.15\lambda$ and 4.28×10^{-4} when $\sigma = 0.25\lambda$. These results suggest that our undersampling approximation works well for a small rough case.

Figure 4 illustrates the total scattering cross section p_c/λ against θ_i the angle of incidence. Due to the Wood

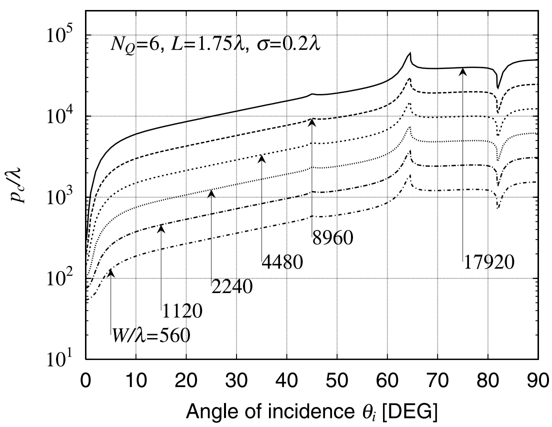

Fig. 4 Total scattering cross section p_c/λ against θ_i the angle of incidence. $L = 1.75\lambda$, $\sigma/\lambda = 0.2$, $W/\lambda = 560$, 1120, 2240, 4480, 8960 and 17920. When $\theta_i > 5°$, p_c increases linearly proportional to W. When $\theta_i \approx 0$, p_c slowly increases as W increases.

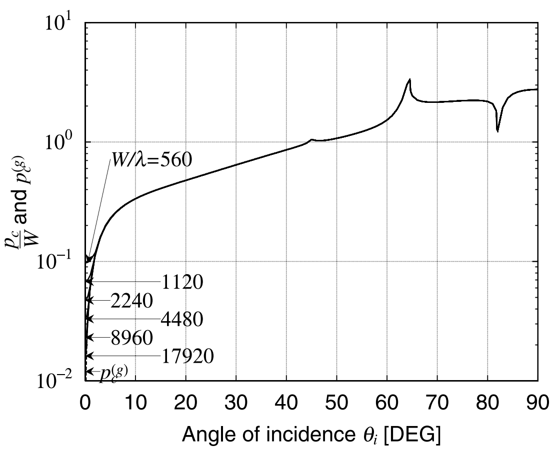

Fig. 5 Total scattering cross section per unit surface p_c/W and $p_c^{(g)}$ against θ_i the angle of incidence. $L = 1.75\lambda$, $\sigma/\lambda = 0.2$, $W/\lambda = 560$, 1120, 2240, 4480, 8960 and 17920. When $\theta_i > 5°$, p_c/W is almost equal to $p_c^{(g)}$.

anomaly, a peak or a dip appeares when θ_i is close to one of the critical angles of incidence. When $\theta_i > 5°$, p_c increases linearly proportional to W. When $\theta_i \approx 0$, however, p_c increases slowly as W increases. These properties of p_c are quite similar to the case of a finite periodic array of rectangular grooves [10].

Figure 5 shows p_c/W and $p_c^{(g)}$ against θ_i. When $\theta_i > 5°$, p_c/W is almost independent of W and is almost equal to $p_c^{(g)}$. Let us see some numerical examples. When θ_i is non-critical and $\theta_i = 30°$, p_c/W is 0.6436086 at $W/\lambda = 1120$ and 0.6435719 at $W/\lambda = 17920$, which are almost equal to $p_c^{(g)} = 0.6435693$. When $\theta_i = 70°$, p_c/W is 2.159498 at $W/\lambda = 1120$ and 2.159427 at $W/\lambda = 17920$, which almost equal $p_c^{(g)} = 2.159422$.

When θ_i is critical and is not grazing, p_c/W slightly depends on W as is shown in Fig. 6. When $\theta_i = 64.62306°$, we have $p_c/W = 2.946957$ at $W/\lambda = 560$ and $p_c/W = 2.945640$ at $W/\lambda = 17920$, which is 0.37% larger than $p_c^{(g)} =$

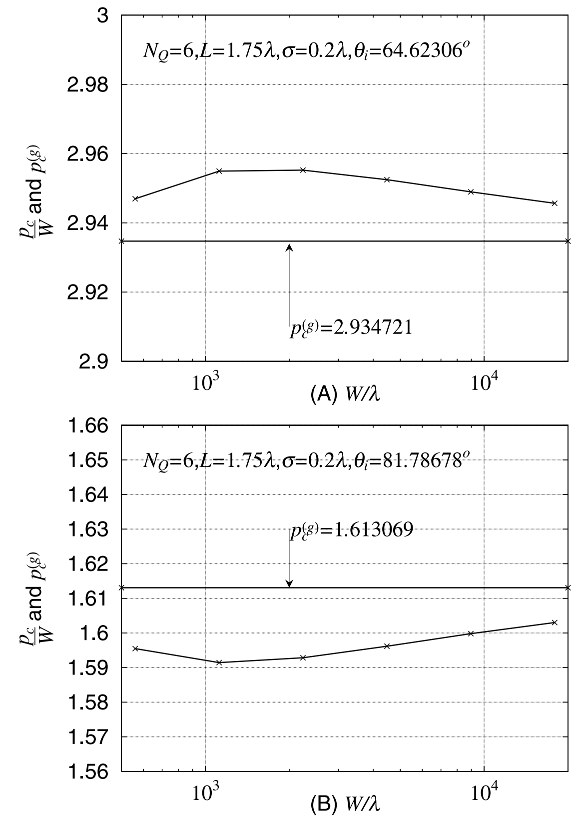

Fig. 6 p_c/W against W/λ. (A) $\theta_i = 64.62306°$ and (B)$\theta_i = 81.78678°$. $L = 1.75\lambda$, $\sigma/\lambda = 0.2$.

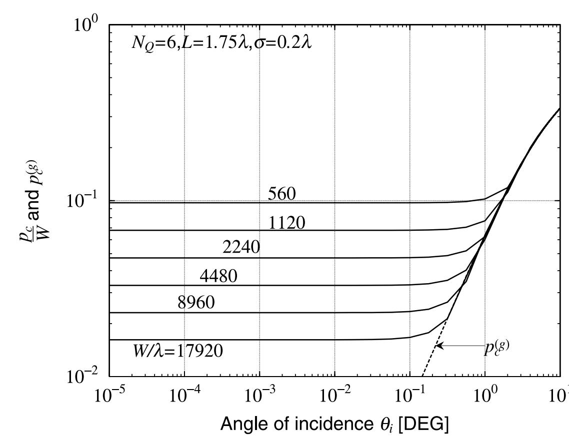

Fig. 7 p_c/W and $p_c^{(g)}$ against small values of θ_i. $L = 1.75\lambda$, $\sigma/\lambda = 0.2$, $W/\lambda = 560$, 1120, 2240, 4480, 8960 and 17920. For a sufficiently small θ_i, p_c/W decreases as W increases.

2.934721. When $\theta_i = 81.78678°$, we find $p_c/W = 1.595484$ at $W/\lambda = 560$ and $p_c/W = 1.603007$ at $W/\lambda = 17920$, which is 0.62% smaller than $p_c^{(g)} = 1.613069$. These results mean that (27) holds accurately even when θ_i is critical, if W is sufficiently large. However, Fig. 6 suggests that p_c/W converges to $p_c^{(g)}$ quite slowly in the case of critical angles.

We illustrate p_c/W and $p_c^{(g)}$ for small θ_i in Fig. 7. We see that p_c/W is almost equal to $p_c^{(g)}$ for $\theta_i > 5°$. As θ_i becomes small, a curve p_c/W branches away from the curve

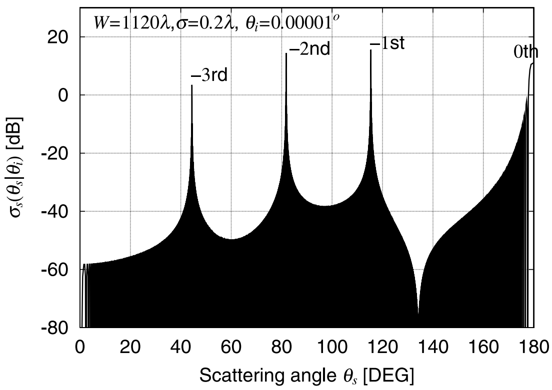

Fig. 8 Scattering cross section $\sigma_s(\theta_s|\theta_i)$ against θ_s. $\theta_i = 0.00001°$ $L = 1.75\lambda$, $\sigma/\lambda = 0.2$.

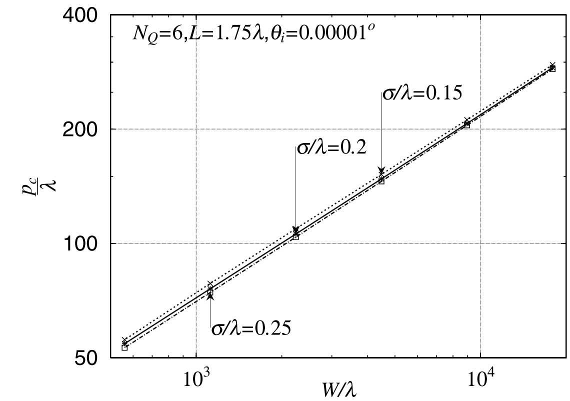

Fig. 10 p_c/λ at $\theta_i = 0.00001°$ against W. $L = 1.75\lambda$, $\sigma/\lambda = 0.15$, 0.2 and 0.25. p_c/λ is almost proportional to $\sqrt{W/\lambda}$.

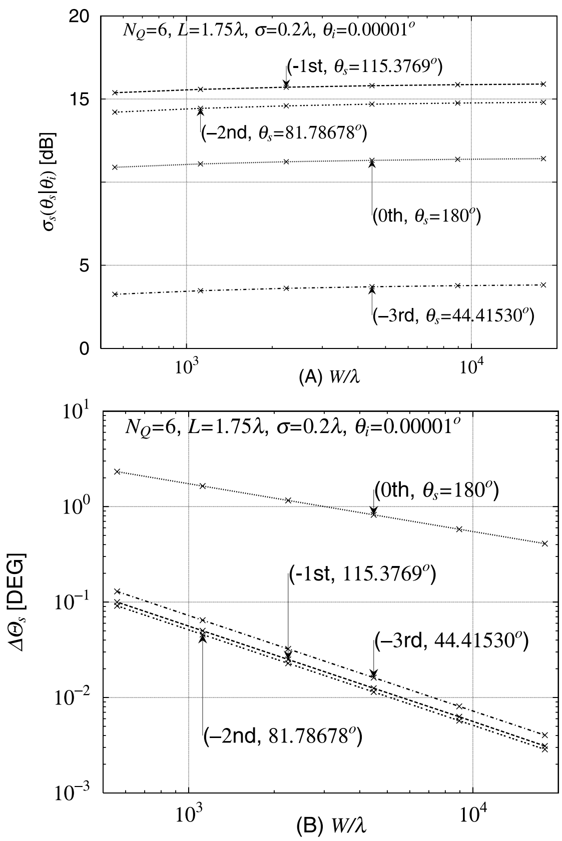

Fig. 9 (A) Peak level and (B) 3 dB beam width $\Delta\Theta_s$ of $\sigma_s(\theta_s|\theta_i)$ against W/λ. $L = 1.75\lambda$, $\sigma/\lambda = 0.2$. The 0th order diffraction beam scattered into the grazing direction $\theta_s = 180°$ has much wide beam width proportional to $\sqrt{\lambda/W}$.

$p_c^{(g)}$ at a branch angle and becomes a flat line. As W gets wider, such a branch angle becomes smaller. Thus, the difference $|p_c/W - p_c^{(g)}|$ becomes much small for a sufficiently large W. We may expect (27) holds for any θ_i when $W \to \infty$. We also see in Fig. 7 that p_c/W decreases with increasing W for a sufficiently small θ_i. This property is quite similar to the case of a finite periodic array of rectangular grooves [10].

Figure 8 shows the differential cross section $\sigma_s(\theta_s|\theta_i)$ against θ_s for $\theta_i = 0.00001°$ and $W/\lambda = 1120$. We see the scattering appears as four peaks at $\theta_s = \Theta_0 = 180°$, $\Theta_{-1} = 115.3769°$, $\Theta_{-2} = 81.78678°$ and $\Theta_{-3} = 44.41530°$, which represent mainlobes of the 0th, −1st, −2nd and −3rd diffraction beams, respectively, whereas the sidelobes appear as hills with a valley at $\theta_s \approx 134°$. These peak levels are plotted against W/λ in Fig. 9(A). The peak levels slightly increase as W gets large but almost saturate when $W/\lambda \geq 8960$. From numerical data of the scattering cross section, we determined the (real) beam width $\Delta\Theta_s$ in Fig. 9(B), which agrees with the estimated value by (18) within 2% errors. Figure 9(B) shows that the 0th order diffraction beam scattered into a grazing direction $\theta_s = 180°$ has a much wide beam width proportional to $\sqrt{\lambda/W}$, whereas other diffraction beams have beam widths proportional to λ/W.

When $W/\lambda = 17920$, for example, the 0th order diffraction beam is $\Delta\Theta_s = 0.409427°$ in beam width, which is about 130 times wider than $\Delta\Theta_s = 0.00310624°$ for the −1st order one at $\theta_s = \Theta_{-1} = 115.3769°$. On the other hand, the peak level of the 0th one is about 4.6 dB lower than that of the −1st one. By these facts and (13), p_{inc} is mainly determined by the 0th order diffraction beam and is roughly proportional to $p_c/\lambda = p_{inc}/\lambda \sim W/\lambda \times \sigma_s(\pi|0)\sqrt{\lambda/W} \sim \sigma_s(\pi|0)\sqrt{W/\lambda}$ for a sufficiently large W. This is an important result of this paper.

We plot p_c/λ against W/λ at $\theta_i = 0.00001°$ in Fig. 10, where p_c is almost proportional to $\sqrt{W/\lambda}$, as is expected above. Furthermore, this figure shows a remarkable fact such that the dependence of p_c on σ is quite small when $W/\lambda = 17920$[†]. Numerically, we have $p_c/\lambda = 295.2657$, 290.2994 and 287.8817 at $\sigma/\lambda = 0.15$, 0.20 and 0.25, respectively, when $\theta_i = 0.00001°$ and $W/\lambda = 17920$.

Let us estimate the asymptotic behavior of p_c for a

[†]Only when W is sufficiently large, p_c/λ at LGLI becomes proportional to $\sqrt{W/\lambda}$ and almost independent of σ. When W is not large, however, p_c/λ depends on σ [8] and p_c/W is almost independent of W at LGLI [9]. Thus, the little dependence on σ should be understood as an asymptotic behavior of p_c when $W \to \infty$.

large W by putting $p_c/\lambda = a + b\sqrt{W/\lambda}$[†]. Determining a and b by numerical data, we obtain

$$\frac{p_c}{\lambda} = \begin{cases} 5.863150 + 2.168190\sqrt{W/\lambda}, & \sigma = 0.15\lambda \\ 4.380363 + 2.138166\sqrt{W/\lambda}, & \sigma = 0.2\lambda \\ 3.002485 + 2.129169\sqrt{W/\lambda}, & \sigma = 0.25\lambda \end{cases}, \tag{30}$$

where a factor in front of $\sqrt{W/\lambda}$ slightly depends on the roughness σ.

The relation (30) means that p_c diverges at LGLI in the weak sense. Here, the divergence in the weak sense means that p_c diverges and p_c/W vanishes, namely,

$$\lim_{W\to\infty} \frac{p_c}{\lambda} = \infty, \tag{31}$$

$$\lim_{W\to\infty} \frac{p_c}{W} = 0. \tag{32}$$

Such the weak divergence of p_c should be understood as a multiple scattering effect. When $W \to \infty$ and the surface (1) approaches to the perfectly sinusoidal surface, p_c diverges and hence has no physical significance. In other words, the scattering can not be well defined for a target with infinite extent. In such a limit, however, p_c/W is well defined and p_c/W vanishes at LGLI. Since p_c/W is expected to equal $p_c^{(g)}$ at LGLI when $W \to \infty$, we may conclude that the diffraction by a sinusoidal surface does not occur at LGLI but the scattering may take place at LGLI if W is finite.

5. Conclusions

At LGLI, no diffraction takes place by a periodic Neumann surface and the scattering may occur if the periodic surface is finite in extent. To solve such contradiction, this paper studied the scattering of a TM plane wave from a perfectly conductive sinusoidal surface with finite extent. By use of the undersampling approximation, we numerically calculated the total scattering cross section p_c for several corrugation widths. We newly found that the total scattering cross section p_c increases almost proportional to the square root of the corrugation width W at LGLI. Then, we may estimate that the total scattering cross section p_c must diverge in the weak sense. Thus, we may conclude that our contradiction at LGLI is caused by such the weak divergence of the total scattering cross section.

The undersampling approximation is not an exact method of analysis. It starts with a physical assumption such that the scattered wave in the far regions is approximated by a finite sum of the diffraction beams, where effects of the edges at $x = \pm W/2$ are neglected implicitly. However, our numerical results have several desirable properties as follows. The scattering cross section per unit surface p_c/W is almost equal to the diffraction cross section $p_c^{(g)}$ for any non-critical angle of incidence. For a critical or grazing angle of incidence p_c/W takes a value close to $p_c^{(g)}$ and approaches to $p_c^{(g)}$ when W becomes large. Furthermore, numerical experiments show that the error with respect to the optical theorem is reasonably small for any angle of incidence, if $W/\lambda > 10^2$ and if the slope parameter $2\pi\sigma/L$ is less than about 2×0.448, where 0.448 is the Rayleigh limit [9], [11]. Therefore, we consider that the undersampling approximation gives reliable results for the far-field properties in the case of a finite sinusoidal surface. To check the validly of the approximation, however, we need the comparison with results by other method. Since there is no other source analyzing a wide case with $W/\lambda > 10^4$, this problem is left for future study.

On the other hand, we have studied an analytical method to obtain the asymptotic behavior of p_c as $W \to \infty$ [17]. However, multiple scattering processes yielding the weak divergence of the total scattering cross section at LGLI are still not clear. This problem, therefore, is left for future study.

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[†]In Ref. [10], we studied the case of a finite periodic array of rectangular grooves, where the maximum number of grooves was restricted to 891 due to the limitation of computer resources. When $(2N_g+1) \leq 891$, p_c/λ was not proportional to $\sqrt{2N_g+1}$ in general and hence we put $p_c/\lambda = a(2N_g+1)^\alpha$ to estimate the asymptotic behavior. However, Fig. 3 in Ref. [10] suggests that p_c/λ becomes proportional to the square root of the corrugation width $\sqrt{W/\lambda} = \sqrt{(2N_g+1)L/\lambda}$ asymptotically, if the number of grooves $(2N_g+1)$ becomes much larger than 891.

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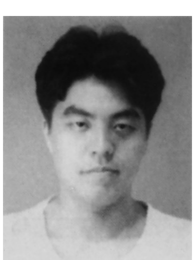

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