LETTER

Diffraction Amplitudes from Periodic Neumann Surface: Low Grazing Limit of Incidence (II)

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SUMMARY The diffraction of a transverse magnetic (TM) plane wave by a perfectly conductive surface made up of a periodic array of rectangular grooves is studied by the modal expansion method. It is found theoretically that the reflection coefficient approaches −1 but no diffraction takes place when the angle of incidence reaches a low grazing limit. Such singular behavior is shown analytically to hold for any finite values of the period, groove depth and groove width and is then demonstrated by numerical examples.

key words: *wave diffraction, periodic array of rectangular grooves, modified diffraction amplitudes, singular behavior*

1. Introduction

Low-grazing-angle scattering from rough surfaces has received much interest in the study of sea echo observation by a ground-based radar [1]. This problem is theoretically interesting because the scattering from a slightly rough Neumann surface becomes singular at a low grazing limit of incidence (LGLI). From a random or periodic surface, no scattering takes place but the reflection coefficient becomes −1 at the LGLI, whereas the reflection coefficient is always 1 in the case of a flat surface. Such a singular behavior is predicted by several multiple scattering theories [2]–[6] but cannot be explained by a single scattering theory [7].

In a previous paper [8], which is referred to as I, however, we introduced the modified diffraction amplitude, which gives us a simple mathematical way to obtain such a singular behavior for any periodic Neumann surface, without using the complicated formulations of multiple scattering. However, these works [2]–[6], [8] were all restricted to a slightly rough case and no theoretical discussions were given for a very rough case. We proposed in I that, regardless of the surface roughness and surface shape, the reflection coefficient must be −1 at the LGLI for any periodic or homogeneous random Neumann surface. However, our proposition is difficult to prove. As a special case of a very rough surface, this paper deals with diffraction by the periodic array of rectangular grooves shown in Fig. 1. Using the modal expansion method [9], we find theoretically that our proposition is true. This theoretical result is verified by numerical examples.

Manuscript received April 5, 2006.
Manuscript revised May 15, 2006.

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DOI: 10.1093/ietele/e89–c.9.1362

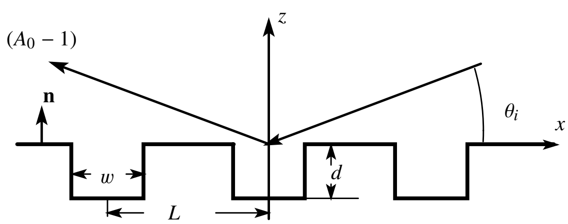

Fig. 1 Diffraction of TM plane wave by periodic array of rectangular grooves. L is the period, and w and d are the width and depth of a groove, respectively. $(A_0 - 1)$ is the reflection coefficient and θ_i is the angle of incidence.

2. Diffraction by Periodic Grooves

Let us consider the diffraction of a TM plane wave by the periodic surface of rectangular grooves shown in Fig. 1. We denote the y component of the magnetic field by $\Psi(x, z)$, which satisfies

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right]\Psi(x, z) = 0, \tag{1}$$

in free space above the periodic surface, where $k = 2\pi/\lambda$ is wavenumber and λ is wavelength. On the periodic surface we have the Neumann condition

$$\frac{\partial}{\partial n}\Psi(x, z) = 0, \tag{2}$$

where n is normal. In the following, we denote the wave function in the region $z \geq 0$ by $\Psi_1(x, z)$ and the wave field inside the grooves by $\Psi_2(x, z)$. To simplify notation, we write

$$k_L = \frac{2\pi}{L}, \quad k_w = \frac{\pi}{w}, \tag{3}$$

where L is the period and w is the width of a groove.

Owing to the periodicity of the surface, $\Psi_1(x, z)$ may take the Floquet form and be written as

$$\Psi_1(x, z) = e^{-ipx - i\beta_0 z} + \sum_{m=-\infty}^{\infty}(A_m - \delta_{m,0})e^{-i(p+mk_L)x+i\beta_m z}, \tag{4}$$

where the first term on the right-hand side is the incident plane wave, and p and β_m are given by

$$p = k\cos(\theta_i), \tag{5}$$

$$\beta_m = \sqrt{k^2 - (p + mk_L)^2},$$
$$Re[\beta_m] \geq 0, \quad Im[\beta_m] \geq 0, \quad (m = 0, \pm 1, \pm 2, \cdots), \tag{6}$$

Here, θ_i is the angle of incidence, and Re and Im denote the real and imaginary parts, respectively. $(A_m - \delta_{m,0})$ is the diffraction amplitude of the mth-order Floquet mode and $(A_0 - 1)$ is the reflection coefficient. A_m is the modified diffraction amplitude introduced in I and is a key idea in this paper.

By the modal expansion method [9], we write $\Psi_2(x,z)$ as a sum of guided waves,

$$\Psi_2(x,z) = \sum_{n=-\infty}^{\infty} u(x-nL)e^{-ipnL} \sum_{l=0}^{\infty} D_l \times \cos[lk_w(x+w/2-nL)]\cos[\gamma_l(z+d)], \quad (7)$$

where d is the groove depth, D_l and γ_l are the amplitude and propagation constant of the lth guided mode, respectively, and $u(x)$ is a rectangular pulse:

$$\gamma_l = \sqrt{k^2 - l^2k_w^2}, \quad (l = 0, 1, 2, \cdots), \quad (8)$$

$$u(x) = \begin{cases} 1, & |x| \le w/2 \\ 0, & |x| > w/2 \end{cases}. \quad (9)$$

Let us determine A_m and D_l using the boundary conditions at $z = 0$. Because of the periodic nature of the problem, we may write the boundary conditions over one period as

$$\Psi_1(x, 0) = \Psi_2(x, 0), \quad (|x| \le w/2), \quad (10)$$

$$\left.\frac{\partial\Psi_1(x,z)}{\partial z}\right|_{z=0} = \begin{cases} \left.\dfrac{\partial\Psi_2(x,z)}{\partial z}\right|_{z=0}, & (|x| \le w/2) \\ 0, & (w/2 < |x| \le L/2) \end{cases}. \quad (11)$$

Substituting (4) and (7) into (10) and (11) yields

$$e^{-ipx}\left[1 + \sum_{m=-\infty}^{\infty}(A_m - \delta_{m,0})e^{-imk_Lx}\right] = \sum_{l=0}^{\infty} D_l \cos[lk_w(x+w/2)]\cos(\gamma_l d), \quad (|x| \le w/2), \quad (12)$$

$$e^{-ipx}[-i\beta_0 + i\sum_{m=-\infty}^{\infty}\beta_m(A_m - \delta_{m,0})e^{-imk_Lx}] = \begin{cases} -\sum_{l=0}^{\infty}\gamma_l D_l \cos[lk_w(x+w/2)]\sin(\gamma_l d), & (|x| \le w/2), \\ 0, & (w/2 < |x| \le L/2) \end{cases}. \quad (13)$$

Next, we multiply (13) by $e^{i(p+mk_L)x}$ and integrate the result over one period. We also multiply (12) by $\cos[nk_w(x+w/2)]$ and integrate the result over the region $-w/2 \le x \le w/2$. Then, we obtain a set of equations for A_m and D_n:

$$iL\beta_m A_m + \sum_{n=0}^{\infty}\gamma_n c_n(p + mk_L)\sin(\gamma_n d)D_n = 2iL\beta_0\delta_{m,0}, \qquad (m = 0, \pm1, \pm2, \cdots), \quad (14)$$

$$\sum_{m=-\infty}^{\infty} c_n(-p - mk_L)A_m - \frac{w\cos(\gamma_n d)}{2}(1 + \delta_{n,0})D_n = 0, \qquad (n = 0, 1, 2, \cdots). \quad (15)$$

Here, $c_n(q)$, $(n = 0,\ 1,\ 2, \cdots)$, is given by

$$c_n(q) = \int_{-w/2}^{w/2}\cos[nk_w(x + w/2)]e^{iqx}dx. \quad (16)$$

Note that the right-hand side of (15) is zero and that of (14) is proportional to β_0, which is similar to the slightly rough case in I.

Let us solve (14) and (15) by truncation. For sufficiently large integers N_A and N_D, we assume

$$A_m = 0, \quad (|m| > N_A), \qquad D_n = 0, \quad (n > N_D), \quad (17)$$

which implies that (14) becomes $(2N_A + 1)$ equations and (15) becomes $(N_D + 1)$ equations. We consider a linear equation system made up of such truncated (14) and (15) for determining the $(2N_A + N_D + 2)$-vector solution $[\{A_m\}, \{D_n\}]^t$, t being the transpose, and we implicitly assume the system has a unique solution. Since the equation system has the excitation term given by $2iL\beta_0\delta_{m,0}$ in (14), the vector solution $[\{A_m\}, \{D_n\}]^t$ is proportional to $2iL\beta_0$. This means that all modified diffraction amplitudes decrease proportionally to β_0 at the LGLI. Also, $\Psi_2(x,z)$ decreases proportionally to β_0. Thus, at the LGLI with $\beta_0 = k\sin(\theta_i) \to 0$, the diffraction amplitude $(A_m - \delta_{m,0})$ is

$$\lim_{\beta_0\to 0}(A_m - \delta_{m,0}) = -\delta_{m,0}, \quad (18)$$

which is the most important result of this paper. We emphasize that (18) holds for any finite values of the period L, depth d and width $w(< L)$. Equation (18) means that the refection coefficient $(A_0 - 1)$ becomes -1 but no diffraction takes place at the LGLI, if the surface is perfectly periodic. This fact suggests that, if the surface is not perfectly periodic, scattering may take place at the LGLI. In other words, imperfections of periodic surfaces could be detected by scattering at the LGLI. Also, note that the idea of the modified diffraction amplitudes gives a simple way to obtain (18), without using the complicated formulations of multiple scattering.

3. Numerical Examples and Conclusions

Let us see some numerical examples for a very rough case. Putting

$$L = 1.5\lambda, \quad w = \frac{L}{2}, \quad d = \pi\lambda, \qquad N_A = 6, \quad N_D = 20, \quad (19)$$

we solve the truncated (14) and (15) numerically. The modal expansion method gives a reasonable solution and the energy error is less than 10^{-7} for any angle of incidence from $\theta_i = 0.00001°$ to 90° in the case of (19). Figure 2 shows the amplitude and phase of diffraction amplitude. When $\theta_i < 0.1°$, the phase $\mathrm{Arg}(A_m - \delta_{m,0})$ is almost constant and $|A_m|$ with $m \ne 0$ decreases proportionally to $\beta_0 = k\sin(\theta_i) \approx k\theta_i$, as expected. Since $|A_0 - 1|$ is almost

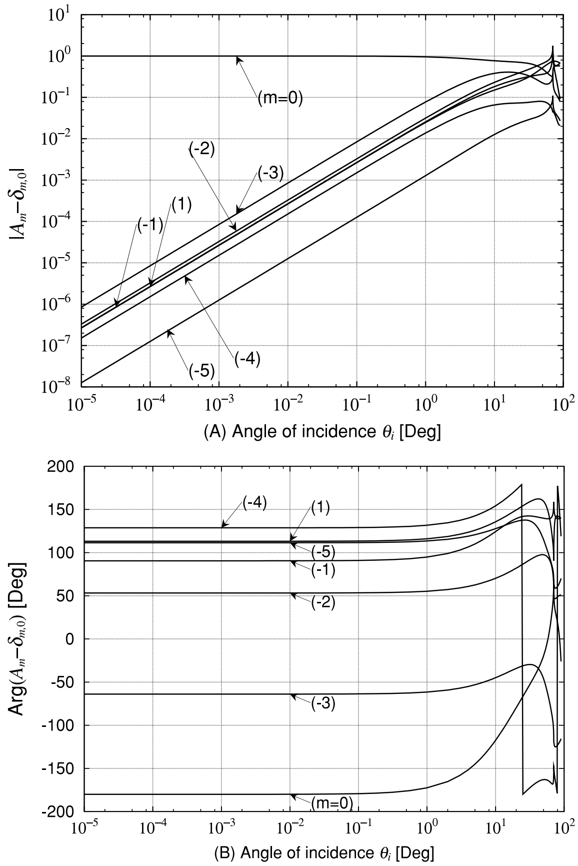

Fig. 2 Amplitude (A) and phase (B) of diffraction amplitude against the angle of incidence θ_i.

1 and Arg$(A_0 - 1)$ is almost equal to $-180°$, the reflection coefficient $(A_0 - 1)$ approaches -1 when $\theta_i < 0.1°$, which supports our proposition. Figure 3 shows $|\Psi_2(x,z)|$ at $x = 0$, which fluctuates along the z axis, because $\Psi_2(x,z)$ is a sum of standing waves. We see that $|\Psi_2(x,z)|$ decreases proportionally to θ_i, as is expected theoretically.

Using several different values of L, w and d, we calculated the diffraction amplitudes numerically. We always found that our proposition is true at the LGLI. By the modal expansion method, we have analyzed another example, diffraction by a triangular grating. In this case, we also found analytically and numerically that, at the LGLI, the reflection coefficient approaches -1 and any other diffraction amplitudes decrease proportionally to θ_i.

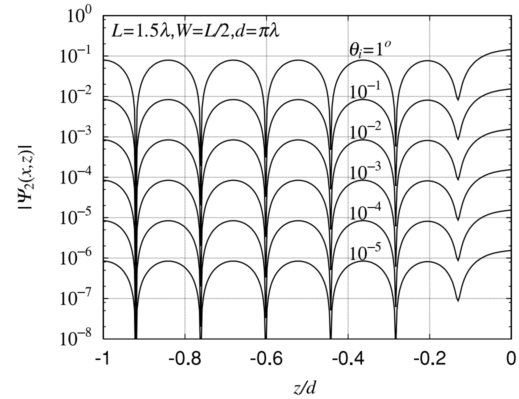

Fig. 3 Field intensity $|\Psi_2(x,z)|$ at $x = 0$ against z. Field intensity decreases proportionally to θ_i the angle of incidence.

With these theoretical and numerical discussions on these special cases, we conclude that our proposition must hold generally even for a very rough periodic Neumann surface. However, clarification of the multiple scattering processes to give (18) and a mathematical proof for our proposition in the general case are still required.

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