LETTER

Wave Scattering from a Periodic Surface with Finite Extent: A Periodic Approach for TM Wave

Junichi NAKAYAMA[†a)], ***Regular Member***, **Toyofumi MORIYAMA**[†], ***Nonmember***, ***and*** **Jiro YAMAKITA**[††], ***Regular Member***

SUMMARY A periodic approach introduced previously is applied to the TM wave scattering from a finite periodic surface. A mathematical relation is proposed to estimate the scattering amplitude from the diffraction amplitude for the periodic surface, where the periodic surface is defined as a superposition of surface profiles generated by displacing the finite periodic surface by every integer multiple of the period Λ. From numerical examples, it is concluded that the scattering cross section for the finite periodic surface can be well estimated from the diffraction amplitude for a sufficiently large Λ.

key words: *wave scattering and diffraction, finite periodic surface, periodic grating*

1. Introduction

The wave scattering from a finite periodic surface has received much interest, because it is related with important applications such as diffraction gratings, leaky wave antenna and waveguide couplers. Several methods of analysis have been introduced by authors [1]–[6]. As an idea of analysis, however, we introduced a periodic approach, which was successfully applied to the TE wave case [7]. This paper deals with an application of the periodic approach to the TM wave scattering from a sinusoidal surface with finite extent.

In the periodic approach, we first generate the periodic surface by translating the finite periodic surface by every integer multiple of a distance Λ and superimposing the translated ones. Then, we calculate the diffracted wave by such a periodic surface with the period Λ. When the period Λ goes to infinity, such a periodic surface becomes the finite periodic surface and hence the diffracted wave physically becomes the scattered wave from the finite periodic surface. Therefore, the scattered wave from the finite periodic surface may be well estimated from the diffracted wave for a sufficiently large Λ. The estimation is done by a mathematical formula connecting the scattering amplitude and diffraction amplitude. We propose here such a mathematical formula suitable for the TM wave incidence.

Manuscript received May 8, 2001.

[†]The authors are with the Faculty of Engineering and Design, Kyoto Institute of Technology, Kyoto-shi, 606-8585 Japan.

[††]The author is with the Faculty of Information Engineering, Okayama Prefecture University, Soja-shi, 719-1197 Japan.

a) E-mail: nakayama@dj.kit.ac.jp

For several values of the period Λ, we numerically calculate the diffraction amplitude, from which the scattering amplitude and the scattering cross section are estimated. It is demonstrated that the scattering amplitude is almost independent of Λ when Λ is large. Then, we conclude that the scattering cross section for the finite periodic surface can be well estimated from the diffraction amplitude for a sufficiently large Λ.

2. Diffraction by Periodic Surface

Let us start with a sinusoidal surface with finite extent:

$$z = f(x) = \begin{cases} \sigma_h \cdot \sin(k_L x), & |x| \le W/2 \\ 0, & |x| > W/2 \end{cases}, \tag{1}$$

$$k_L = \frac{2\pi}{L}, \tag{2}$$

where σ_h is the corrugation height, W is the width of periodic corrugation and L is the period. Translating $f(x)$ by every integer multiple of Λ and superimposing the translated ones, we obtain the periodic surface as

$$z = f_p(x) = f_p(x+\Lambda) = \sum_{n=-\infty}^{\infty} f(x+n\Lambda), \tag{3}$$

where $\Lambda \gg W$ is implicitly assumed.

We write the y component of the magnetic field by $\psi(x,z)$, which satisfies the wave equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right]\psi(x,z) = 0, \tag{4}$$

in free space and the boundary condition

$$\frac{\partial}{\partial n}\psi(x,z) = 0, \qquad z = f_p(x), \tag{5}$$

on the periodic surface (3). Here, $\partial/\partial n$ is the normal derivative, $k = 2\pi/\lambda$ is the wave number and λ is the wavelength. We write the magnetic field as

$$\psi(x,z) = e^{-ipx}e^{-i\beta(p)z} + e^{-ipx}e^{+i\beta(p)z} + \psi_d(x,z,\Lambda), \tag{6}$$

where the first term in the right hand side is the incident plane wave, the second is the specularly reflected wave, and $\psi_d(x,z,\Lambda)$ is the diffracted wave due to the

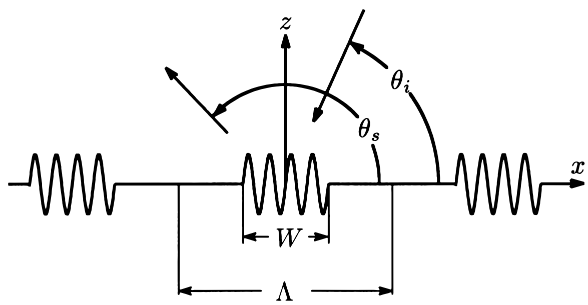

Fig. 1 Scattering and diffraction of a plane wave from a periodic surface with the period Λ. θ_i is the angle of incidence and θ_s is a scattering angle. W is the corrugation width of a partially periodic surface. When the period Λ goes to infinity, the periodic surface becomes a finite periodic surface.

corrugation (3). Here, p and $\beta(p)$ are given by

$$p = k \cdot \cos\theta_i, \tag{7}$$

$$\beta(p) = \sqrt{k^2 - p^2}, \quad \mathrm{Im}\,[\beta(p)] \geq 0, \tag{8}$$

where θ_i is the angle of incidence (See Fig. 1) and Im stands for the imaginary part.

Since the surface (3) is periodic, the diffracted wave $\psi_d(x, z, \Lambda)$ may have the Floquet form:

$$\psi_d(x, z, \Lambda) = \frac{e^{-ipx}}{2\pi} \sum_{m=-\infty}^{\infty} k_\Lambda \cdot a(mk_\Lambda, \Lambda) \times e^{-imk_\Lambda x + i\beta(p+mk_\Lambda)z}, \tag{9}$$

where $k_\Lambda \cdot a(mk_\Lambda, \Lambda)/(2\pi) = a(mk_\Lambda, \Lambda)/\Lambda$ is the wave amplitude diffracted into the direction θ_m given by

$$\cos(\theta_m) = -\cos(\theta_i) - \frac{m}{k} k_\Lambda, \qquad k_\Lambda = \frac{2\pi}{\Lambda}. \tag{10}$$

In what follows, we regard $a(mk_\Lambda, \Lambda)$ is the sample value of $a(s, \Lambda)$ at $s = mk_\Lambda$, whereas $a(s, \Lambda)$ for any s will be estimated by interpolation from the sample sequence $\{a(mk_\Lambda, \Lambda),\ m = 0, \pm 1, \pm 2, \cdots\}$. Note that (9) becomes an integral when $k_\Lambda = 2\pi/\Lambda \to 0$.

Since the periodic surface and free space are lossless, the energy conservation law holds:

$$\frac{1}{2\pi\beta(p)} \sum_{m=-\infty}^{\infty} \mathrm{Re}[\beta(p + mk_\Lambda)] |a(mk_\Lambda, \Lambda)|^2 k_\Lambda + 2\mathrm{Re}[a(0, \Lambda)] = 0, \tag{11}$$

where Re stands for the real part.

3. Estimation of the Scattering Amplitude from the Diffraction Amplitude

When $\Lambda \to \infty$, the periodic surface $f_p(x)$ becomes the finite sinusoidal surface $f(x)$, and the diffracted wave $\psi_d(x, z, \Lambda)$ becomes the scattered wave from the finite sinusoidal surface $f(x)$. As $k_\Lambda = 2\pi/\Lambda \to 0$, the Floquet form (9) is reduced to an integral representation of the scattered wave [7]

$$\psi_s(x, z) = \lim_{\Lambda \to \infty} \psi_d(x, z, \Lambda) = \frac{e^{-ipx}}{2\pi} \int_{-\infty}^{\infty} a(s, \infty) e^{-isx + i\beta(p+s)z} ds, \tag{12}$$

where $a(s, \infty) = \lim_{\Lambda \to \infty} a(s, \Lambda)$ is the scattering amplitude.

On the other hand, the diffraction amplitude $a(mk_\Lambda\ , \Lambda)\ /\Lambda$ is physically expected to be proportional to $1/\Lambda$, because the number of finite corrugations per unit length is $1/\Lambda$. Therefore, $a(s, \Lambda)$ is almost independent of Λ when Λ is large enough and wave interactions between finite corrugations become negligibly small. This means that the scattering amplitude $a(s, \infty)$ may be well approximated by $a(s, \Lambda)$ when Λ is large.

To estimate $a(s, \Lambda)$ from the diffraction amplitude, we proposed an interpolation formula [7], which works well only for TE wave case. For the TM case, however, we introduce another interpolation formula:

$$\beta(p + s) a(s, \Lambda) = \sum_{m=-\infty}^{\infty} \beta(p + mk_\Lambda) a(mk_\Lambda, \Lambda) \frac{\sin\left[\frac{s - mk_\Lambda}{2}\Lambda\right]}{\left[\frac{s - mk_\Lambda}{2}\Lambda\right]}. \tag{13}$$

By use of (13), we obtain $\psi_s(x, z, \Lambda)$ an approximate expression for the scattered wave,

$$\psi_s(x, z, \Lambda) = \frac{e^{-ipx}}{2\pi} \int_{-\infty}^{\infty} a(s, \Lambda) e^{-isx + i\beta(p+s)z} ds. \tag{14}$$

The interpolation formula (13) may be derived as follows. Expressions (9) and (14) are valid in a region with $z \geq \sigma_h = \max\{f(x)\}$. Assuming that $\psi_d(x, z, \Lambda)$, $\psi_s(x, z, \Lambda)$, and their derivatives may be calculated from (9) and (14) even at $z = 0$, however, we put

$$\left.\frac{\partial \psi_s(x, z, \Lambda)}{\partial z}\right|_{z=0} = \begin{cases} \left.\frac{\partial \psi_d(x,z,\Lambda)}{\partial z}\right|_{z=0}, & |x| < \Lambda/2 \\ 0, & |x| > \Lambda/2 \end{cases}, \tag{15}$$

of which Fourier transform yields (13).

From (14) we obtain the energy conservation law:

$$\frac{kW}{2\pi\beta(p)} \int_0^\pi \sigma(\theta_s|\theta_i) d\theta_s + 2\mathrm{Re}[a(0, \Lambda)] = 0, \tag{16}$$

$$\sigma(\theta_s|\theta_i) = \frac{k^2 \sin^2(\theta_s)}{kW} |a(-k\cos\theta_i - k \cdot \cos\theta_s, \Lambda)|^2, \tag{17}$$

where $\sigma(\theta_s|\theta_i)$ is the scattering cross section and θ_s is a scattering angle (See Fig. 1).

We have expected that $a(s, \Lambda)$ is almost independent of Λ. To evaluate this numerically, we introduce

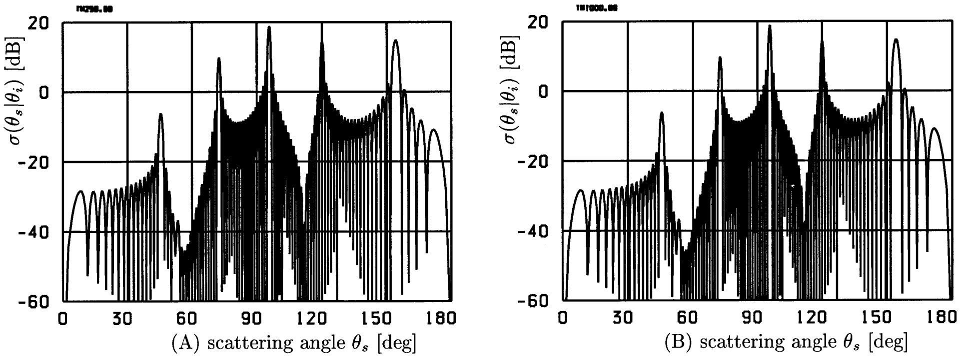

Fig. 2 Scattering cross section $\sigma(\theta_s|\theta_i)$ estimated from the diffraction amplitude $\{a(mk_\Lambda, \Lambda)\}$ by interpolation. $\theta_i = 60°$, $W = 20L$, $L = 2.5\lambda$, $\sigma_h = 0.1\lambda$. (A) $\Lambda = 250\lambda$, (B) $\Lambda = 1000\lambda$.

the distance $\rho(\Lambda, \Lambda')$ between $a(s, \Lambda)$ and $a(s, \Lambda')$ as

$$\rho(\Lambda, \Lambda') = \frac{||a(s,\Lambda) - a(s,\Lambda')||}{\sqrt{||a(s,\Lambda)|| \cdot ||a(s,\Lambda')||}}, \tag{18}$$

$$||a(s,\Lambda)||^2 = \frac{1}{k}\int_{-\infty}^{\infty} |\beta(p+s)a(s,\Lambda)|^2 ds, \tag{19}$$

where the weighted norm $||a(s,\Lambda)||$ is easily evaluated in terms of $\{a(mk_\Lambda, \Lambda)\}$ by (13).

4. Numerical Results and Conclusions

For numerical calculations we put

$$L = 2.5\lambda, \quad W = 20L, \quad \sigma_h = 0.1\lambda. \tag{20}$$

Four different values of the period Λ are considered: $\Lambda = 125\lambda$, 250λ, 500λ and 1000λ. By the method of Green's theorem in Ref. [8], we determine $\{a(mk_\Lambda, \Lambda)/\Lambda\}$ with an energy error less than 0.5%, where $\{a(mk_\Lambda, \Lambda)/\Lambda\}$ is approximated by a $(4\Lambda/\lambda+1)$-dimensional vector. From $\{a(mk_\Lambda, \Lambda)\}$, we estimate the amplitude $a(s, \Lambda)$ by the interpolation formula (13) to obtain the scattering cross section $\sigma(\theta_s|\theta_i)$ illustrated in Fig. 2. In case of Fig. 2 with $\theta_i = 60°$, there are major peaks at scattering angles 45.6°, 72.5°, 95.7°, 120.00° and 154.2°, which are effects of diffraction by the periodic corrugation (1) with the period $L = 2.5\lambda$. We see that Fig. 2(A) for $\Lambda = 250\lambda$ agrees well with Fig. 2(B) for $\Lambda = 1000\lambda$. We also calculated the norm $||a(s,\Lambda)||$ and the distance $\rho(\Lambda, \Lambda')$ numerically for $\theta_i = 60°$. The norm $||a(s,\Lambda)||$ slightly decreases when Λ increases: $||a(s,\Lambda)||$ is 29.5018 for $\Lambda = 125\lambda$, 29.4912 for $\Lambda = 250\lambda$, 29.4904 for $\Lambda = 500\lambda$ and 29.4900 for $\Lambda = 1000\lambda$. We found that the distances are also small: $\rho(125\lambda, 1000\lambda) = 1.045 \times 10^{-3}$, $\rho(250\lambda, 1000\lambda) = 3.411 \times 10^{-4}$, and $\rho(500\lambda, 1000\lambda) = 1.983 \times 10^{-4}$. For other angles of incidence, numerical calculations were carried out also. Then we found that, in case of (20), $a(s,\Lambda)$ and $\sigma(\theta_s|\theta_i)$ can be well estimated by the interpolation (13) if the period Λ is larger than 250λ.

We have applied the periodic approach introduced in Ref. [7] to the TM wave scattering from a finite periodic surface. We propose an interpolation formula to estimate the scattering amplitude for the finite periodic surface from the diffraction amplitude. From numerical examples, it is concluded that the scattering cross section for the finite periodic surface can be well estimated from the diffraction amplitude for a sufficiently large Λ.

The periodic approach may be applied to the electromagnetic wave scattering from a dielectric periodic surface with finite extent, if the interpolation formula is modified appropriately. However, a rigorous mathematical treatment and conditions under which the periodic approach works are still open question. These problems are left for future study.

References

[1] D. Maystre, "Rigorous theory of light scattering from rough surfaces," J. Optics, Paris, vol.15, no.1, pp.43–51, 1984.

[2] M. Tomita, "Thin-film waveguide with a periodic groove structure of finite extent," J. Opt. Soc. Am. A., pp.1455–1469, 1989.

[3] K. Kobayashi and T. Eizawa, "Plane wave diffraction by a finite sinusoidal grating," IEICE Trans., vol.E74, pp.2815–2826, 1991.

[4] D.C. Skigin, V.V. Veremey, and R. Mittra, "Superdirectiveity radiation from finite gratings of rectangular grooves," IEEE Trans., vol.AP-42, pp.376–383, 1999.

[5] J. Nakayama, "Periodic Fourier transform and its application to wave scattering from a finite periodic surface," IEICE Trans. Electron., vol.E83-C, no.3. pp.481–487, 2000.

[6] A. Neto, S. Maci, G. Vecchi, and M. Sabbadini, "A truncated Floquet wave diffraction method for the full wave analysis of large phased arrays, Part 1: Basic principles and 2-D cases," IEEE Trans. Antennas & Propag., vol.48, no.3, pp.594–600, 2000.

[7] J. Nakayama, T. Moriyama, and J. Yamakita, "Wave scattering from a periodic surface with finite extent: A periodic approach," IEICE Trans. Electron., vol.E84-C, no.8, pp.1111–1113, Aug. 2001.

[8] J.A. DeSanto, "Scattering from a perfectly reflecting arbitrary periodic surface: An exact theory," Radio Science, vol.16, no.6, pp.1315–1326, 1981.